第五章

特徵擷取技術

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各章學習目標

- 第五章
  - 瞭解各種特徵擷取方法
  - 學習如何實作各種特徵擷取技術

第五章 特徵擷取技術

- 統計特徵(Statistical Features)
- 形狀特徵(Shape Features)
- 紋理特徵(Texture Features)
- 關聯特徵(Relational Features)
- Feature and Image Classification

Feature Extraction

- After segmentation, specific features representing the characteristics and properties of the segmented regions in the image need to be computed for object classification and understanding.
- There are four major categories of features for region representation:
  - Statistical Features
    - Provide quantitative information about the pixels within a segmented region.
    - Ex: Histogram, Moments, Energy, Entropy, Contrast, Edges

Image Analysis: Feature Extraction

- Shape Features
  - Provide information about the characteristic shape of the region boundary.
  - Ex: Boundary encoding, Moments, Hough Transform, Region Representation, Morphological Features
- Texture Features
  - Provide information about the local texture within the region or the corresponding part of the image.
  - Ex: second-order histogram statistics, co-occurrence matrix, wavelet processing.
- Relational Features
  - Provide information about the relational and hierarchical structure of the regions associated with a single or a group of objects.

Statistical Pixel-Level Features

- The *histogram* of the gray values of pixels

$$p(r_i) = \frac{n(r_i)}{n}$$

- *Mean* of the gray values of the pixels

$$m = \frac{1}{n}\sum_{i=0}^{L-1} r_i\, p(r_i)$$

- *Varianc*e and central moments in the region

$$\mu_n = \sum_{i=0}^{L-1} p(r_i)(r_i - m)^n$$

where n=2 is the variance of the region.
n=3 is a measure of noncentrality
n=4 is a measure of flatness of the histogram.

Statistical Pixel-Level Features

- *Energy*: Total energy of the gray-values of pixels

$$E = \sum_{i=0}^{L-1} \left[p(r_i) \right]^2$$

- Entropy 熵

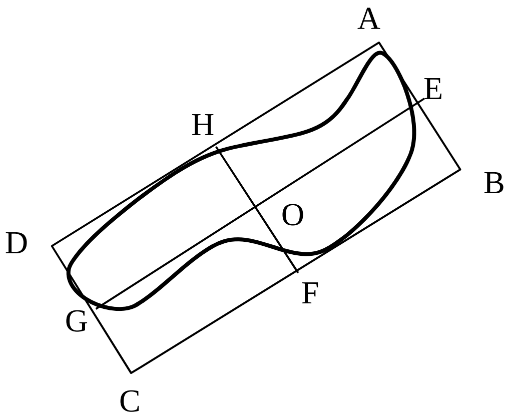
$$Ent = \sum_{i=0}^{L-1} p(r_i) \log_2 (r_i)$$

- Local contrast
- $$C(x,y) = \frac{\left| P_c(x,y) - P_s(x,y) \right|}{\max\{P_c(x,y), P_s(x,y)\}}$$
- Maximum and minimum gray values

Shape Features

- The shape of a region is defined by the spatial distribution of boundary pixels.
  - Circularity, compactness, and elongatedness through the minimum bounded rectangle that covers the region.
  - Several features using the boundary pixels of the segmented region can be computed as
    - Chain code for boundary contour
    - Fourier descriptor of boundary contour
    - Central moments based shape features for segmented region
    - Morphological shape descriptors

Some Shape Features



- Longest axis *GE*.
- Shortest axis *HF*.
- Perimeter and area of the minimum bounded rectangle *ABCD*.
- Elongation ratio: *GE/HF*
- Perimeter *p* and area *A* of the segmented region.

- Circularity

$$C = \frac{4\pi A}{p^2}$$

- Compactness

$$C_p = \frac{p^2}{A}$$

Boundary Encoding :Chain Code

- Define a neighborhood matrix with the orientation primitives with respect to the center pixel.
- The code of specific orientation are set for 8-connected neighborhood directions.
- The orientation directions are codes with a numerical value ranging from 0 to 7.
- The boundary contour needs to be approximated as a list of segments that have pre-selected length and directions.

Boundary Encoding :Chain Code

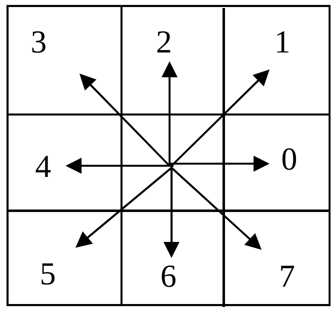
- To obtain boundary segments representing a piecewise approximation of the original boundary contour, the “divide and conquer ” is applied.
  - Selects two points on a boundary contour as vertices.
  - A straight line joining the two selected vertices can be used to approximate the respective curve segment if it satisfies a “maximum-deviation” criterion for no further division of the curve segment.
    - The maximum deviation criterion is based on the perpendicular distance between any point on the original curve segment between the selected vertices and corresponding approximated straight-line segment.

Boundary Encoding :Chain Code

  - If the perpendicular distance or deviation of any point on the curve segment from the approximated straight-line segment exceeds a pre-selected deviation threashold, the curve segment is further divided at the point of maximum deviation.
  - This process of dividing the segments with additional vertices continues until all approximated straight-line segments satisfy the maximum-deviation criterion.
- The representation is further approximated using the orientation primitive of the 8-connected neighborhood.
- Two parameters can change the chain code: number of orientation primitives and the maximum deviation threshold used in approximating the curve.

Boundary Encoding :Chain Code

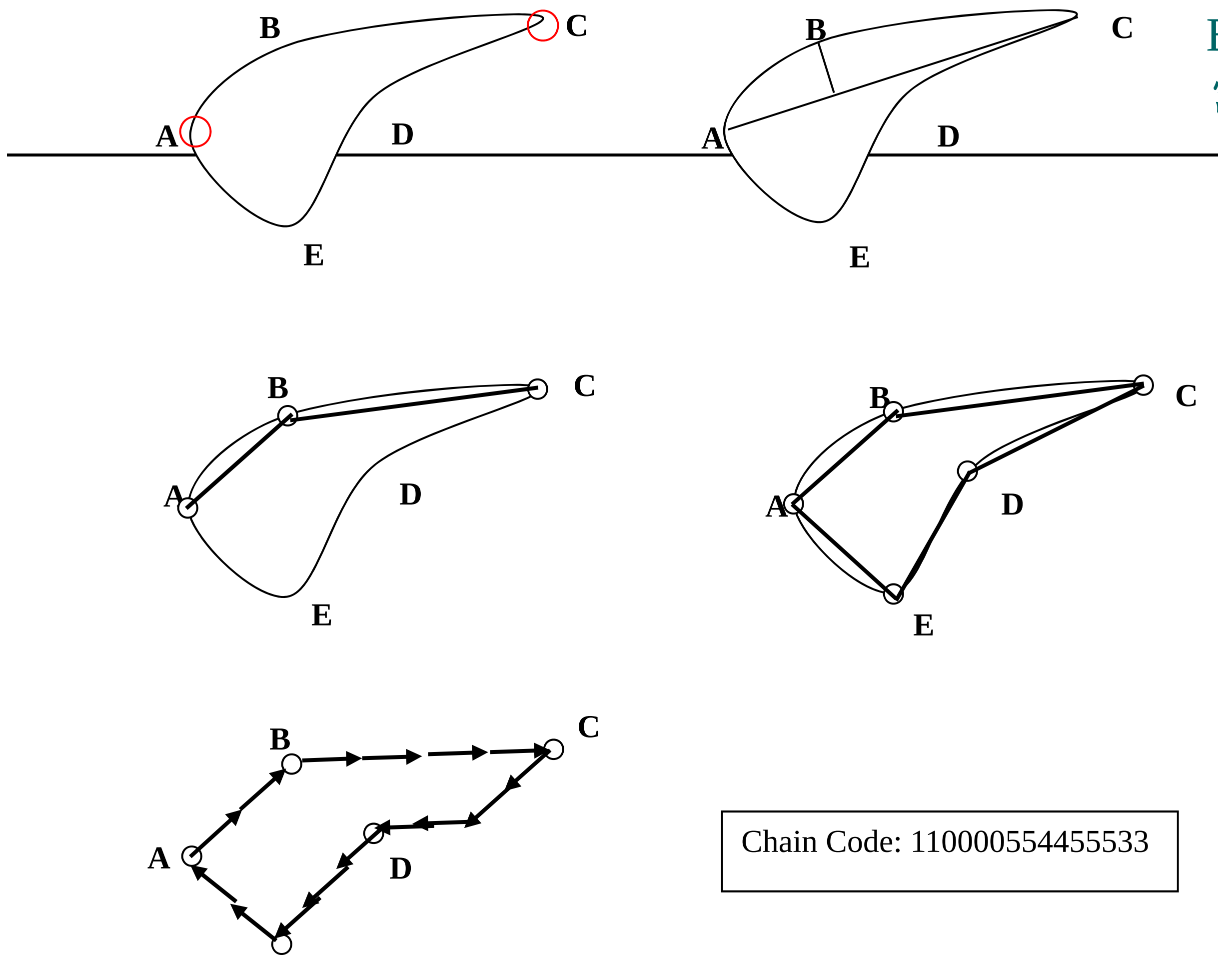
3	2	1
4	x_c	0
5	6	7



- The 8-connected neighborhood codes (left) and the orientation directions (right) with respect to the center pixel *xc*.

選取在方向及梯度上有較明顯的兩的頂點為起始點

BF大於預設值，
需將AC分成AB, BC



- A schematic example of developing chain code for a region with boundary contour *ABCDE.* From top left to bottom right: the original boundary contour, two points *A* and *C* with maximum vertical distance parameter *BF*, two segments *AB* and *BC* approximating the contour *ABC*, five segments approximating the entire contour *ABCDE*, contour approximation represented in terms of orientation primitives, and the respective chain code of the boundary contour.

Boundary Encoding: Fourier Descriptor

- Fourier series may be used to approximate a closed boundary of a region.
  - Assume that the boundary of an object is expressed as a sequence of N points with the coordinates $u[n]=\{x(n), y(n)\}$, such that

$$u(n) = x(n) + iy(n) \qquad n = 0, 1, 2, \ldots, N-1$$

  - The discrete Fourier Transform of the sequence $u[n]$ is the Fourier descriptor $F_d[n]$ of the boundary and is defined as

$$F_d[n] = \frac{1}{N}\sum_{n=0}^{N-1} u(n) e^{-2\pi i n/N} \qquad for\ 0 \le n \le N-1$$

Boundary Encoding: Fourier Descriptor

- Rigid geometric transformation of a boundary such as *translation*, *rotation* and *scaling* can be represented by simple operations on its Fourier transform.
- The Fourier descriptors can be used as shape descriptors for region matching dealing with translation, rotation and scaling.

Moments for Shape Description

- The shape of a boundary or contour can be represented quantitatively by the central moments for matching.
- The central moments represent specific geometrical properties of the shape and are invariant to the translation, rotation and scaling.
- The central moments μ_{pq} of a segmented region or binary image $f(x,y)$ are given by

$$\mu_{pq} = \sum_{i=1}^{L}\sum_{j=1}^{L}(x_i - \bar{x})^p (y_j - \bar{y})^q f(x,y)$$

$$\bar{x} = \sum_{i=1}^{L}\sum_{j=1}^{L} x_i f(x_i, y_j)$$

$$\bar{y} = \sum_{i=1}^{L}\sum_{j=1}^{L} y_j f(x_i, y_j)$$

Moments for Shape Description

- The normalized central moments are defined as

$$\eta_{pq} = \frac{\mu_{pq}}{(\mu_{00})^{\gamma}} \qquad \gamma = \frac{p+q}{2} + 1$$

- There are seven invariant moments for shape matching

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

$$\phi_3 = (\eta_{20} - \eta_{02})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

$$\phi_5 = (\eta_{20} - 3\eta_{12})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]$$

$$\phi_6 = (\eta_{20} - \eta_{02})\left[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$$

$$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] + (3\eta_{12} - \eta_{03})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]$$

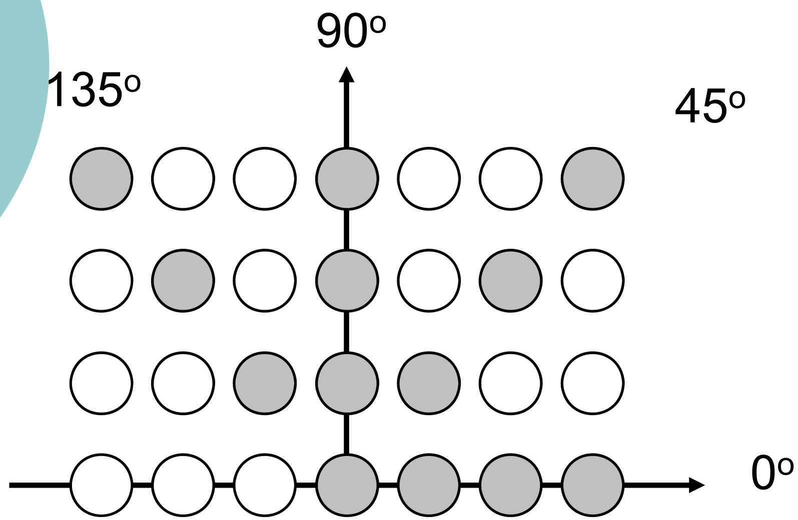
Texture Features

- Texture is an important spatial property .
- There are three major approaches to represent texture
  - Statistical
    - Based on region histograms, their extensions and their moments.
    - Representing the high-order distribution of gray values in the image are used for texture representing.
  - Structural
    - Arrangements of pre-specified primitives in texture representation, such as a repetitive arrangement of square and triangular shapes.
  - Spectral
    - Based on the autocorrelation function of a region or on the power distribution in Fourier transform domain.
    - Texture is represented by a group of specific spatio-frequency components, such as Fourier and wavelet transform.

Texture Features

- Gray-level co-occurrence matrix (GLCM)
  - Exploits the high-order distribution of gray values of pixels that are defined with a specific distance or neighborhood criterion.
  - GLCM $P(i,j)$ is the distribution of the number of occurrences of a pair of gray values i and j separated by a distance vector $d=[dx, dy]$
  - The GLCM can be normalized by dividing each value in the matrix by the total number of occurences providing the probability of occurrence of a pair of gray values separated by a distance vector.
- Statistical texture features are computed from the normalized GLCM.
  - The second-order histogram $H(y_q, y_r, d)$ representing the probability of occurrence of a pair of gray values y_q and y_r separated by a distance vector d.

Gray Level Co-occurrence Matrix (GLCM)



The four direction for the GLCM

1	1	2	2
1	1	2	2
3	3	1	1
3	3	1	1

Gray Level	1	2	3
1	2	2	0
2	0	1	0
3	2	1	1

Co-occurrence matrix for 45°

Gray Level Co-occurrence matrix (GLCM)

- Normalized Co-occurrence matrix

Gray level image

$$H(y_q, y_r, d) = \frac{Num(f(x,y) = y_q \cap f(x+\Delta m, y+\Delta n) = y_r)}{\sum_{m,n=1}^{M,N} pairs}$$

- 其中，*Num*表示影像中滿足*f*(*x* ,*y*)的總點對數目，又四個角度距離為d的定義分別為0度(*x*+*d*, *y*+*0*)、45度(*x*+*d*, *y*+*d*) 、90度(*x*+*0*, *y*+*d*)和135度(*x*-*d*, *y*+*d*) 。

45 degree (△m=1, △n=1)
5x5 matrix Gray level 0~2

H(1,1,1)

f(x,y) =

0	0	0	1	2
1	1	0	1	1
1	2	2	2	1
2	1	0	2	0
2	0	1	0	1

i \ *j*	0	1	2
0	2	2	0
1	0	4	2
2	2	1	1

H(1,1,1) = *4/14*

Texture Feature

- Entropy of $H(y_q, y_r, d)$
  - The entropy is a measure of texture nonuniformity

$$S_H = -\sum_{y_q=y_1}^{y_t} \sum_{y_r=y_1}^{y_t} H(y_q, y_r, d) \log_{10}[H(y_q, y_r, d)]$$

- Angular Second Moment of $H(y_q, y_r, d)$
  - ASMH indicates the degree of homogeneity among textures

$$ASM_H = \sum_{y_q=y_1}^{y_t} \sum_{y_q=y_1}^{y_t} [H(y_q, y_r, d)]^2$$

- Contrast of $H(y_q, y_r, d)$
  - $\partial(y_q, y_r)$ is a measure of intensity similarity

$$Contrast = \sum_{y_q=y_1}^{y_t} \sum_{y_q=y_1}^{y_t} \partial(y_q, y_r) H(y_q, y_r, d)$$

Texture Feature

- Inverse Difference Moment of $H(y_q, y_r, d)$, IDM_H
  - Provides a measure of the local homogeneity among texture

$$IDM_H = \sum_{y_q=y_1}^{y_t} \sum_{y_r=y_1}^{y_t} \frac{H(y_d, y_r, d)}{1+\partial(y_q, y_r)}$$

- Correlation of $H(y_q, y_r, d)$
  - The correlation attribute is large for similar elements of the second-order histogram.

$$Cor_H = \frac{1}{\sigma_{y_q}\sigma_{y_r}} \sum_{y_q=y_1}^{y_t} \sum_{y_r=y_1}^{y_t} (y_q - \mu y_q)(y_r - \mu y_r) H(y_q, y_r, d)$$

$$H_m(y_q, d) = \sum_{y_q=y_1}^{y_t} H(y_q, y_r, d)$$

$$H_m(y_r, d) = \sum_{y_r=y_1}^{y_t} H(y_q, y_r, d)$$

Texture Feature

- Mean of $H(y_q, y_r, d)$, μ_{Hm}
  - The mean characterizes the nature of the gray-level distribution

$$\mu_{Hm} = \sum_{y_q=y_1}^{y_t} y_q H_m(y_q, d)$$

- Deviation of $H_m(y_q, d)$, δ_{Hm}
  - Indicates the amount of spread around the mean of the marginal distribution.

$$\sigma_{Hm} = \sqrt{\sum_{y_q=y_1}^{y_t} \left[y_q - \sum_{y_r=y_1}^{y_t} y_r H_m(y_r, d) \right]^2 H_m(y_q, d)}$$

- Entropy of $H_d(y_s, d)$, $S_{Hd(ys,d)}$

$$S_{H_d(y_s,d)} = -\sum_{y_s=y_1}^{y_t} H_d(y_s, d) \log_{10}[H_d(y_s, d)]$$

Texture Feature

- Angular Second Moment of $H_d(y_s, d)$, $ASM_{Hd(ys, d)}$

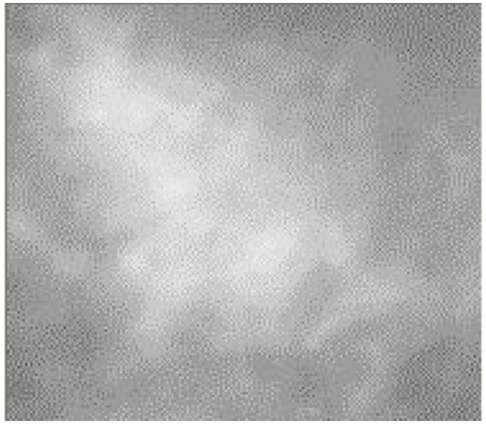
$$ASM_{H_d(y_s,d)} = \sum_{y_s=y_1}^{y_t} [H_d(y_s,d)]^2$$

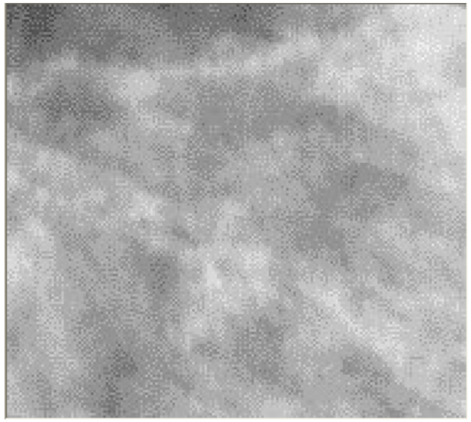
- Mean of $H_d(y_s, d)$, $\mu_{Hd(ys, d)}$,

$$\mu_{H_d(y_s,d)} = \sum_{y_s=y_1}^{y_t} y_s [H_d(y_s,d)]$$

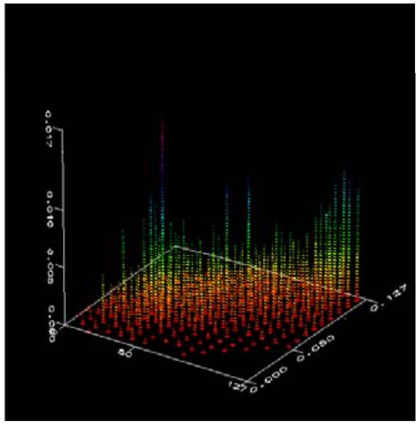
Benign lesion of X-ray mammogram

malignant lesion of X-ray mammogram

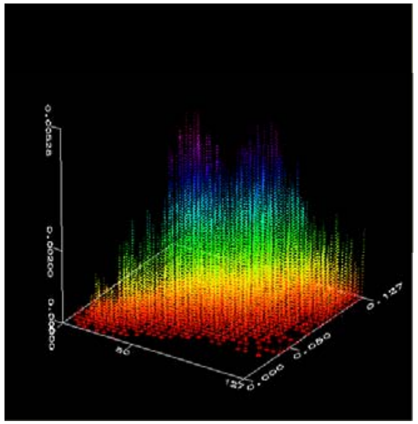




GLCM of Fig. (a)



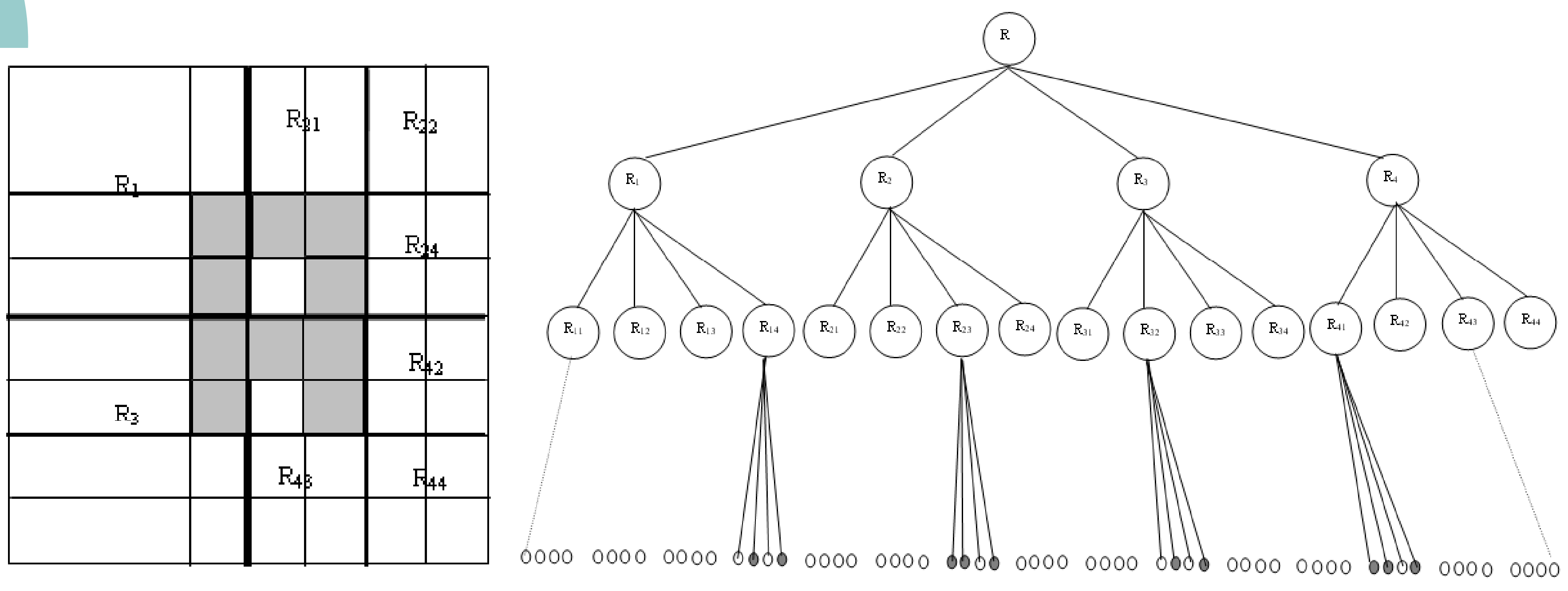
GLCM of Fig. (b)



Relational Features

- Relational features
  - Provide information about adjacencies, repetitive patterns and geometrical relationships among regions of an object.
  - Could be extended to describe the geometrical relationships among objects in an image or a scene.
  - The relational features can be described in the form of graphs or rules using a specific syntax or language
  - The quad-tree based region descriptors can be used for object recognition and classification using the tree matching algorithms.

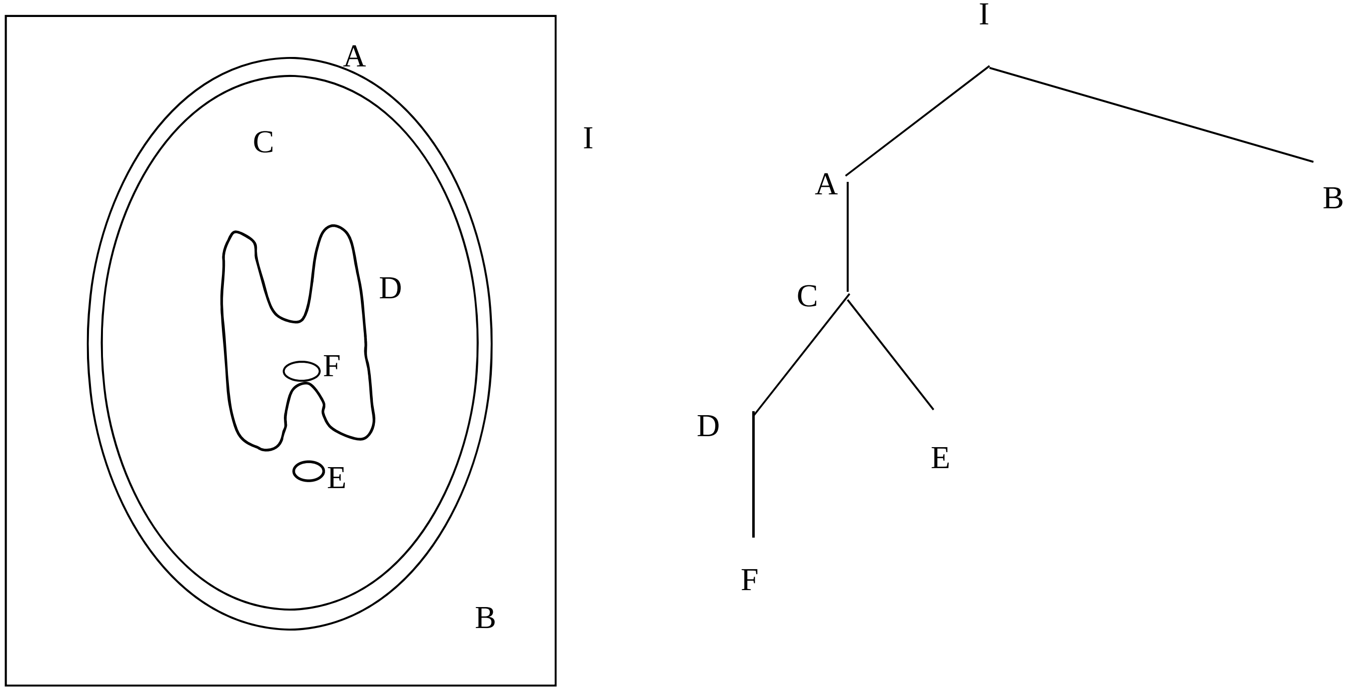
Relational Features



A block representation of an image with major quad partitions (top) and its quad-tree representation.

Relational Features

A tree structure representation of brain ventricles
for applications in brain image segmentation and analysis



Feature and Image Classification

- Features selected for image representation are classified for object recognition and characterization
- Feature Based Pattern Classifiers
  - Statistical Pattern Recognition
    - Unsupervised Learning
    - Supervised Learning
  - Syntactical Pattern Recognition
    - Logical predicates
  - Rule-Based Classifiers
  - Model-Based Classifiers
  - Artificial Neural Networks

Feature and Image Classification

- Statistical Pattern Recognition
  - Unsupervised Learning
    - Cluster the data based on their separation in the feature space.
    - K-means and fuzzy clustering methods
  - Supervised Learning
    - It uses labeled clusters of training samples in the feature space as models of classes.
    - Nearest neighbor classifier, which assigns a data point to the nearest class model in the feature space.

Pixel Classification Through Clustering

- K-means Clustering
  - To partition d-dimensional data into k clusters
  - Initially places k clusters at arbitrarily selected cluster centroids v_i; i=1,2,…,k and modifies centroids for the formation of new cluster shapes optimizing the objective function.

Pixel Classification Through Clustering

The k-means clustering algorithm

- Step 1: Select the number of clusters k with initial cluster centroids v_i; $i=1,2,\ldots,k$
- Step 2: partition the input data points into k clusters by assigning each data point x_j to the closest cluster centroid v_i using the selected distance measure,(Euclidean distance)

$$d_{ij} = \left| x_j - v_i \right|$$

- Step 3: Compute a cluster assignment matrix U representing the partition of the data points with the binary membership value of the jth data point to the ith cluster such that

$$u_{ij} \in \{0,1\} \quad \text{for all } i, j$$

$$\sum_{i=1}^{k} u_{ij} = 1 \quad \text{for all } j \text{ and } 0 < \sum_{j=1}^{n} u_{ij} < n \quad \text{for all } i$$

Pixel Classification Through Clustering

- Step 4: Re-compute the centroids using the membership values as

$$v_i = \frac{\sum_{j=1}^{n} u_{ij} x_j}{\sum_{j=1}^{n} u_{ij}} \quad \text{for all } i$$

- Step 5:If cluster centroids or the assignment matrix does not change from the previous iteration, stop; otherwise go to Step 2.
- The k-means clustering method optimizes the sum-of-squared-error based objective function

$$J_w(U,v) = \sum_{i=1}^{k} \sum_{j=1}^{n} \left\| x_j - v_i \right\|^2$$

K-means演算法

- K-means演算法:
  - Step1:對於多維度的資料集合 $X=\{x_1,x_2,\ldots,x_n\}$，選擇分群的群數k，並初始化每群的中心點 $\{v_1, v_2, \ldots, v_n\}$ 。
  - Step2:利用Euclidean distance公式去計算每群中的資料點 x_i 以求得最接近其中心點 v_i 的資料點。
    而Euclidean distance公式定義為 $d_{ij} = \|x_j - v_i\|$
  - Step3:計算每一群的assignment 矩陣 U，此矩陣表示每個資料點的分佈 $U=\lfloor u_{ij} \rfloor$ 二元關係值，即第i群中第j個資料點的二元關係值表示如下:
    $$u_{ij} \in \{0,1\} \text{ for all } i, j \text{ ; } \sum_{i=1}^{k} u_{ij} = 1 \quad \text{for all } j \text{ and } 0 < \sum_{j=1}^{n} u_{ij} < n \quad \text{for all } i$$
  - Step4:利用membership values去更新中心點 ，更新公式表示為
    $$v_i = \frac{\sum_{j=1} u_{ij} x_j}{\sum_{j=1}^{n} u_{ij}} \quad \text{for all } i$$
  - Step5:如果每群的中心點或assignment matrix跟上一次迭代時的值不再有所變動且維持定值，表示收斂並停止計算;否則回Step2繼續。

Pixel Classification Through Clustering

- Fuzzy c-Means Clustering
  - The k-mean clustering method utilizes the hard binary values for the membership of a data point to the cluster.
  - The fuzzy c-means clustering method utilizes an adaptable membership value that can be updated based using the distribution statistics of the data points assigned to the cluster minimizing the following objective function

$$J_m(U,v) = \sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^m d_{ij}^2 = \sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^m \left\| x_j - v_i \right\|^2$$

  - Fuzziness index can be defined between 1 and very large value for the highest level of fuzziness (maximum allowable variance within a cluster)

Pixel Classification Through Clustering

- The membership values in the fuzzy c-means algorithm can be defined as

$$0 \le u_{ij} \le 1 \quad \text{for all } i, j$$

$$\sum_{i=1}^{c} u_{ij} = 1 \quad \text{for all } j \quad \text{and} \quad 0 \le \sum_{j=1}^{n} u_{ij} < n \quad \text{for all } i$$

Pixel Classification Through Clustering

- The fuzzy c-means algorithm
  - Step 1: Select an initial fuzzy pseudo-partition. i.e., assign values to all u_{ij}.
  - Step 2: Repeat
  - Step 3: Compute the centroid of each cluster using the fuzzy partition.

$$v_i = \frac{\sum_{j=1}^{n} u_{ij}^{m} x_j}{\sum_{j=1}^{n} u_{ij}^{m}}, \quad \text{for all } i = 1,2,\ldots,c$$

  - Step 4: Update the fuzzy partition, ie., the u_{ij}.

$$u_{ij} = \frac{\left\| x_j - v_i \right\|^{-2/(m-1)}}{\sum_{i=1}^{c} \left\| x_j - v_i \right\|^{-2/(m-1)}}$$

  - Step 5: Until there is no difference in the centroids from the previous iteration, or there is not a significant difference in the membership values, the algorithm has converged, stop; otherwise go to Step 2.

Nearest Neighbor Classifier

A distance measure $D_j(\mathbf{f})$ is defined by the Euclidean distance in the feature space as

$$D_j(\mathbf{f}) = \left\| \mathbf{f} - \mathbf{u}_j \right\|$$

where $\mathbf{u}_j = \frac{1}{N_j} \sum_{f \in c_j} \mathbf{f}_j \qquad j = 1,2,\ldots C$

is the mean of the feature vectors for the class c_j and N_j is the total number of feature vectors in the class c_j.

The unknown feature vector is assigned to the class c_i if

$$D_i(\mathbf{f}) = \min_{j=1}^{C}[D_j(\mathbf{f})]$$

Statistical classification Method

- A probabilistic approach can be applied to the task of classification to incorporate *a priori* knowledge to improve performance.
  - Bayesian and maximum likelihood methods have been widely used in object recognition and classification.
- Bayesian

Statistical classification Method

- The probability of a feature vector *f* belonging to the class *i* (c_i) is denoted by $p(c_i / f)$.
- The average risk of wrong classification for assigning the feature vector to the class c_j is defined as

$$r_j(\mathrm{f}) = \sum_{k=1}^{C} Z_{kj}\, p(c_k/\mathrm{f})$$

$$r_j(\mathrm{f}) = \sum_{k=1}^{C} Z_{kj}\, p(\mathrm{f}/c_k) P(c_k)$$

- A Bayes classifier assigns an unknown feature vector to the class c_j if

$$r_i(\mathrm{f}) < r_j(\mathrm{f})$$

$$\sum_{k=1}^{C} Z_{ki}\, p(\mathrm{f}/c_k) P(c_k) < \sum_{q=1}^{C} Z_{qj}\, p(\mathrm{f}/c_q) P(c_q)$$

Feature and Image Classification

- Rule-Based Systems
  - Analyzes the feature vector using multiple sets or rules that are designed to check specific conditions in the database of feature vectors to initiate an action.
  - The rules are composed of two parts
    - Condition premises
    - Actions
  - They are based on expert knowledge to infer the action if the conditions are satisfied.

Feature and Image Classification

- A rule-based system has three sets of rules
  - Supervisory or strategy rules
    - Guide the analysis process and provide the control actions such as starting and stopping analysis.
  - Focus of attention rules
    - Bring specific features into analysis by accessing and extracting the required information or features from the database
  - Knowledge rules
    - Analyze the information with respect to the required conditions and implement an action causing changes in the output database.

A schematic diagram of a rule-based system for image analysis

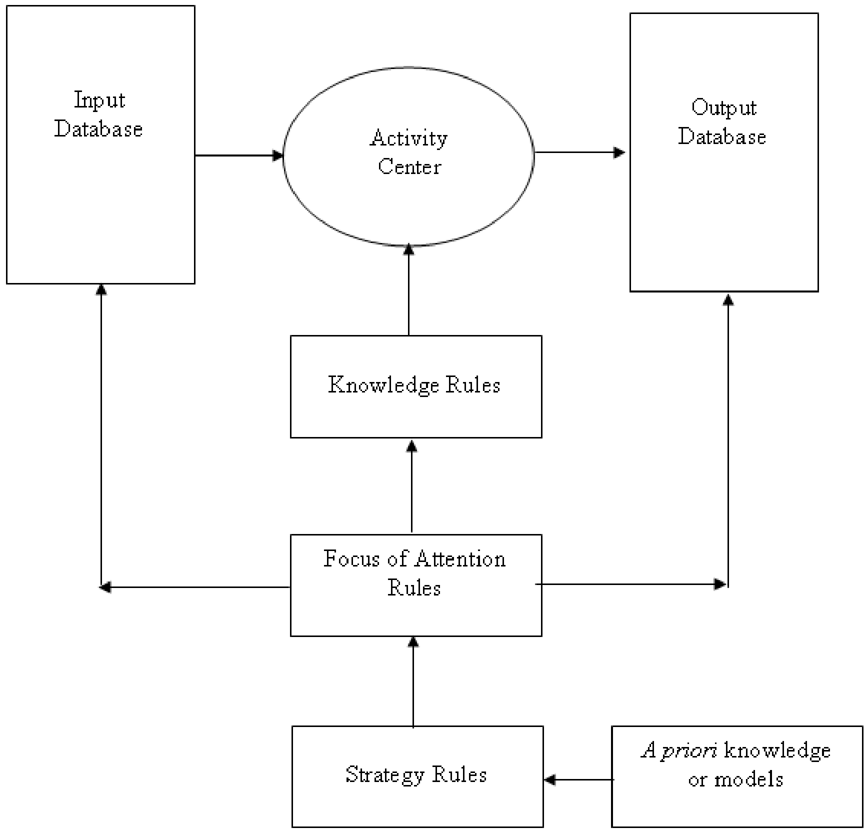


Figure 8.15. A schematic diagram of a rule-based system for image analysis.

Strategy Rules

Strategy Rule SR1:

If

NONE REGION is ACTIVE
NONE REGION is ANALYZED

Then

ACTIVATE FOCUS in SPINAL_CORD AREA

Strategy Rule SR2:

If

ANALYZED REGION is in SPINAL_CORD AREA
ALL REGIONS in SPINAL_CORD AREA are NOT ANALYZED

Then

ACTIVATE FOCUS in SPINAL_CORD AREA

Strategy Rule SR3:

If

ALL REGIONS in SPINAL_CORD AREA are ANALYZED
ALL REGION in LEFT_LUNG AREA are NOT ANALYZED

Then

ACTIVATE FOCUS in LEFT_LUNG AREA

FOA Rules

Focus of Attention Rule FR1:

If

REGION-X is in FOCUS AREA
REGION-X is LARGEST
REGION-X is NOT ANALYZED

Then

ACTIVATE REGION-X

Focus of Attention Rule FR2:

If

REGION-X is in ACTIVE
MODEL is NOT ACTIVE

Then

ACTIVATE KNOWLEDGE_MERGE rules

Knowledge Rules

Knowledge Rule: Merge_Region_KR1

If

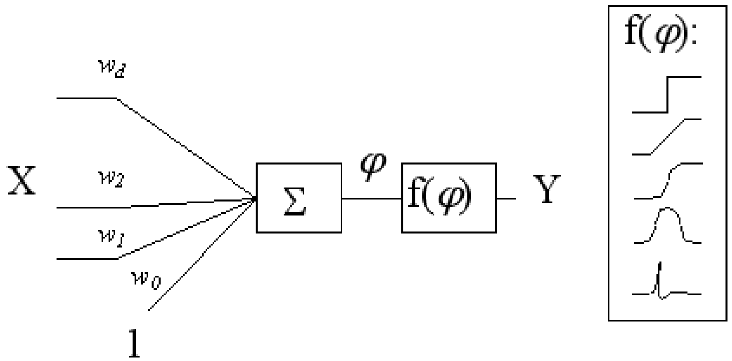
REGION-1 is SMALL
REGION-1 has GIGH ADJACENCY with REGION-2
DIFFERENCE between AVERAGE VALUE of REGION-1 and REGION-2 is LOW or VERY LOW
REGION-2 is LARGE or VERY LARGE

Then

MERGE REGION-1 in REGION-2
PUT_STATUS ANALYZED in REGION-1 and REGION-2

Image and Feature Classification: Neural Networks

- Several neural networks have been used for feature classification for object recognition and image interpretation.
  - Backpropagation
  - Radial Basis Function
  - Associative Memories
  - Self-Organizing Map



$$\varphi = \sum_{t=1}^{d} x_i w_i + w_0$$

Otsu自動二值法演算法(1/2)

- 假設一張影像的灰階值分佈為0到L-1，L為最大的灰階值。令灰階值為i的像素點個數有 x_i 點，且全部像素點個數為n點。則在灰階值為 i 的機率分佈為

$$p_i = \frac{x_i}{n}$$

- 則一張影像的像素點基於一門檻值t可被分為兩個群組: C_0= [$0,1,\ldots,t$]和C_1 = [$t+1, t+2,\ldots, L-1$]。兩個群組各自的機率為 w_0和w_1，其公式表示如下:

$$w_0 = \sum_{i=1}^{t} p_i = w(t) \qquad w_1 = \sum_{i=t+1}^{L} p_i = 1 - w(t)$$

- 則兩個群組的平均值μ_0和μ_1可以被求得如下:

$$\mu_0 = \sum_{i=1}^{t} \frac{i \times p_i}{w_0} \qquad \mu_1 = \sum_{i=t+1}^{L} \frac{i \times p_i}{w_1}$$

Otsu自動二值法演算法(2/2)

- 其變異數 σ_0^2 和 σ_1^2 也可被求得如下: (6)

$$\sigma_0^2 = \sum_{i=1}^{t} \frac{(1-\mu_0)^2 \times P_i}{w_0}$$

(7)

$$\sigma_1^2 = \sum_{i=t+1}^{L} \frac{(1-\mu_1)^2 \times P_i}{w_1}$$

$$\sigma_w^2(t) = w_0\sigma_0^2(t) + w_1\sigma_1^2(t)$$

(8)

- 因此，最佳門檻值 t_{opt} 能藉由最小化within-class的變異數σ_w^2 而求得，如下公式所示：

(9)

$$t_{opt} = \underset{0 \le t < L}{Arg\ Min}\{\sigma_w^2(t)\}$$

Reference

- Rafael C. Gonzalez and Richard E. Woods, Digital Image Processing (2nd, 3rd Edition)
- Atam P. Dhawan, Medical Image Analysis, Wiley Interscience, 2003.