# 第六章

## 形態學計算

國立雲林科技大學 資訊工程研究所
張傳育(Chuan-Yu Chang ) 博士
Office: EB 212
TEL: 05-5342601 ext. 4337
E-mail: chuanyu@yuntech.edu.tw

# Introduction

- Mathematical morphology
  - A tool for extracting image components that are useful in the representation and description of region shape, such as boundaries, skeletons, and convex hull.
- Sets in mathematical morphology represent objects in an image.
- 2D integer space $Z^2$
  - $(x,y)$ coordinates
- $Z^3$: gray-scale digital images
  - $(x,y)$ coordinates, and gray-level value

# Preliminaries

- Let $A$ be a set in $Z^2$, If $a=(a_1, a_2)$ is an element of $A$

  $$a \in A$$

- If $a$ is not an element of $A$, we write

  $$a \notin A$$

- The set with no elements is called the *null* or *empty* set and denoted by the symbol $\phi$ .
- The elements of the sets with which we are concerned are the coordinates of pixels representing objects.
  - Ex:

    $$C = \{ w \mid w = -d, for\ d \in D\}$$

    set $C$ is the set of elements, $w$, such that $w$ is formed by multiplying each of the two coordinates of all the elements of set $D$ by -1.

# Preliminaries

- Basic Concepts from Set Theory
  - Subset
    - If every element of a set $A$ is also an element of another set $B$, then $A$ is said to be a subset of $B$.

$$A \subseteq B$$

  - Union
    - The set of all elements belonging to either $A$, $B$, or both

$$C = A \cup B$$

  - Intersection
    - The set of all elements belonging to both $A$ and $B$

$$D = A \cap B$$

# Preliminaries (cont.)

- Disjoint (mutually excusive)
  - If the two set have no common elements

$$A \cap B = \phi$$

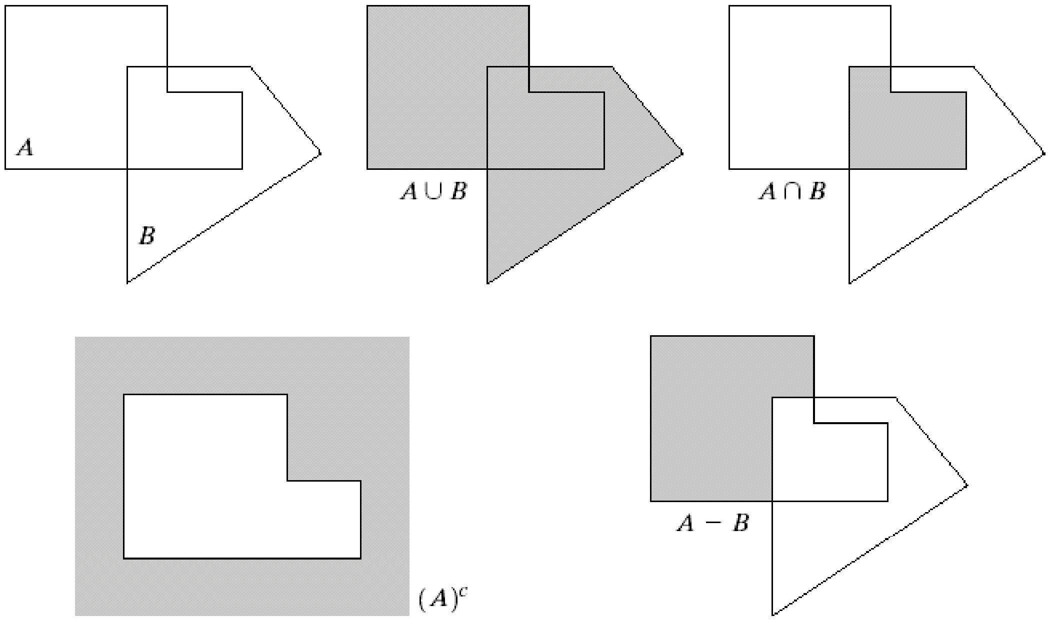
- Complement:
  - The complement of a set $A$ is the set of elements not contained in $A$

$$A^c = \{w \mid w \notin A\}$$

- Difference:
  - the set of elements that belong to $A$, but not to $B$.

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$$

# Preliminaries (cont.)




a b c
d e

**FIGURE 9.1**
(a) Two sets $A$ and $B$. (b) The union of $A$ and $B$. (c) The intersection of $A$ and $B$. (d) The complement of $A$. (e) The difference between $A$ and $B$.

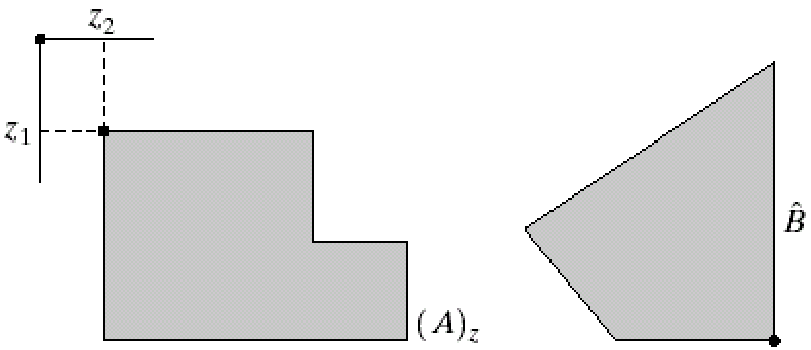
# Preliminaries (cont.)

- Reflection

$$\hat{B} = \{w \mid w = -b, \ for \ \ b \in B\}$$

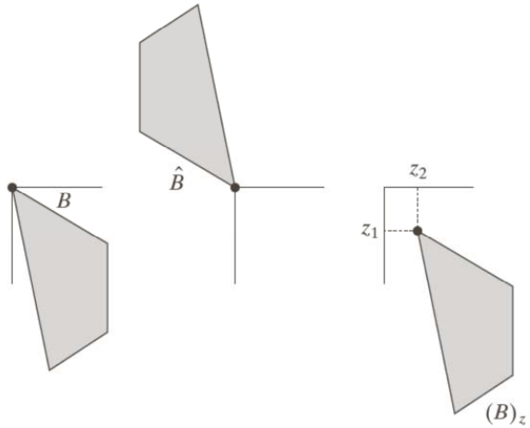
- Translation

$$(A)_z = \{c \mid c = a + z, \ for \ \ a \in A\}$$




a b

**FIGURE 9.2**
(a) Translation of $A$ by $z$.
(b) Reflection of $B$. The sets $A$ and $B$ are from Fig. 9.1.



# Logic Operations Involving Binary Images

- The principal logic operations used in image processing are *AND*, *OR*, and *NOT*
- The three basic logical operations
  - Performed on a pixel by pixel basis between corresponding pixels of two or more images.
  - Logical operation are restricted to binary variables

**TABLE 9.1**
The three basic logical operations.

| $p$ | $q$ | $p$ AND $q$ (also $p \cdot q$) | $p$ OR $q$ (also $p + q$) | NOT ($p$) (also $\bar{p}$) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 |

  - These operations are *functionally complete* in the sense that they can be combined to form any other logic operation

# Logic Operations Involving Binary Images

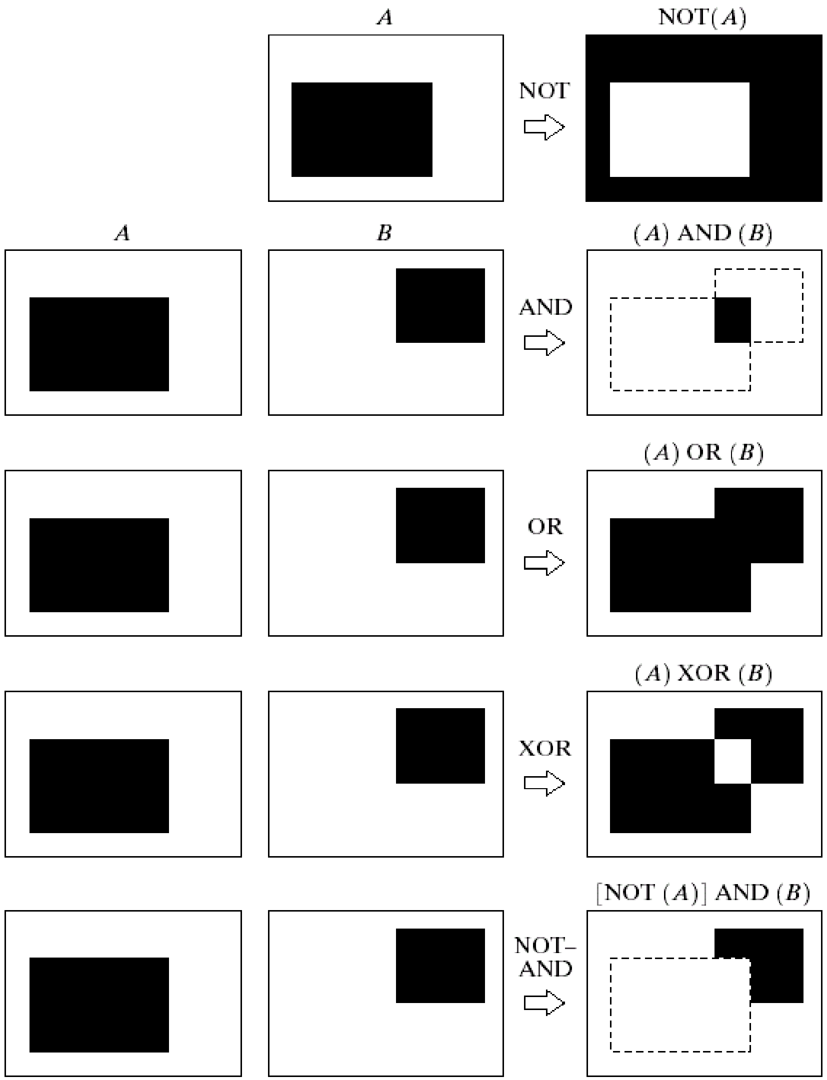



**FIGURE 9.3** Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

–Black indicates a binary 1

–White indicates a 0.

# Dilation and Erosion

- For sets $A$ and $B$ in $Z^2$
- The dilation of $A$ by $B$, denoted

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \phi\}$$
$$= \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

where set $B$ is referred to as the *structuring element*.

- The dilation of $A$ by $B$ is the set of all displacements, $z$, such that and $A$ overlap by at least one element.

$\hat{B}$

# Dilation and Erosion (cont.)

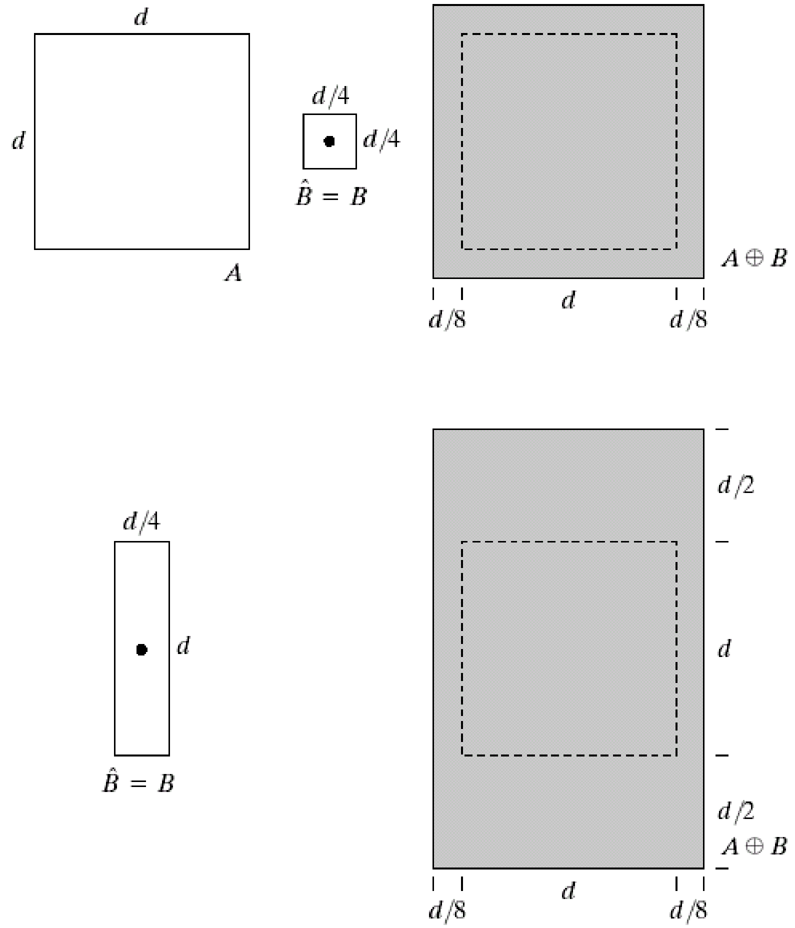
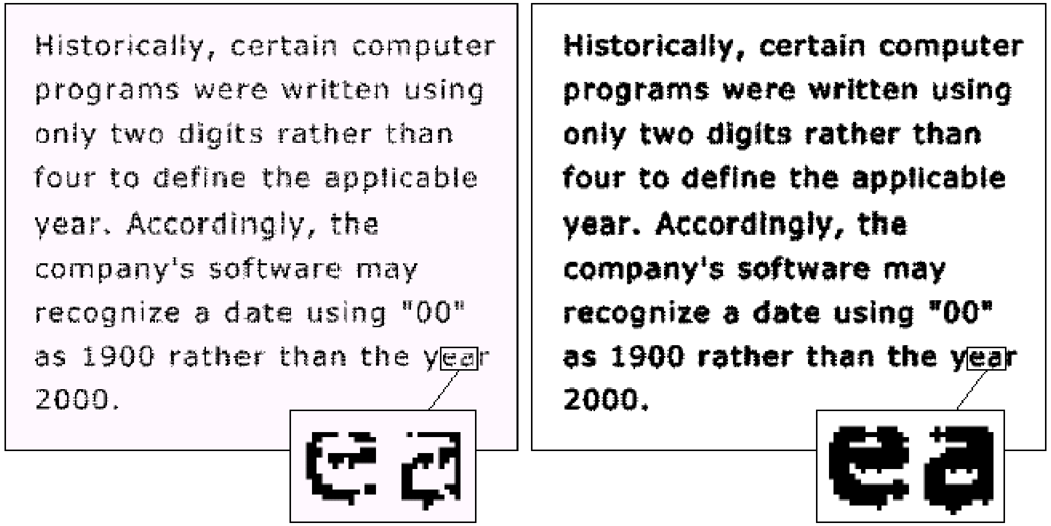



**FIGURE 9.4**
(a) Set $A$.
(b) Square structuring element (dot is the center).
(c) Dilation of $A$ by $B$, shown shaded.
(d) Elongated structuring element.
(e) Dilation of $A$ using this element.

# Dilation and Erosion (cont.)

- Example of dilation
  - bridging gaps
    - The maximum length of the breaks is known to be two pixels.
    - A simple structuring element that can be used for repairing the gaps is shown in Fig. 9.5(b)




| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

a c
b

**FIGURE 9.5** (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

# Dilation and Erosion

- For sets $A$ and $B$ in $Z^2$
- The erosion of $A$ by $B$, denoted $A \Theta B$

$$A \Theta B = \{ z \mid (B)_z \subseteq A \}$$

where set $B$ is referred to as the *structuring element.*

- The erosion of $A$ by $B$ is the set of all points $z$ such that $B$, translated by $z$, is contained in $A$.

# Dilation and Erosion

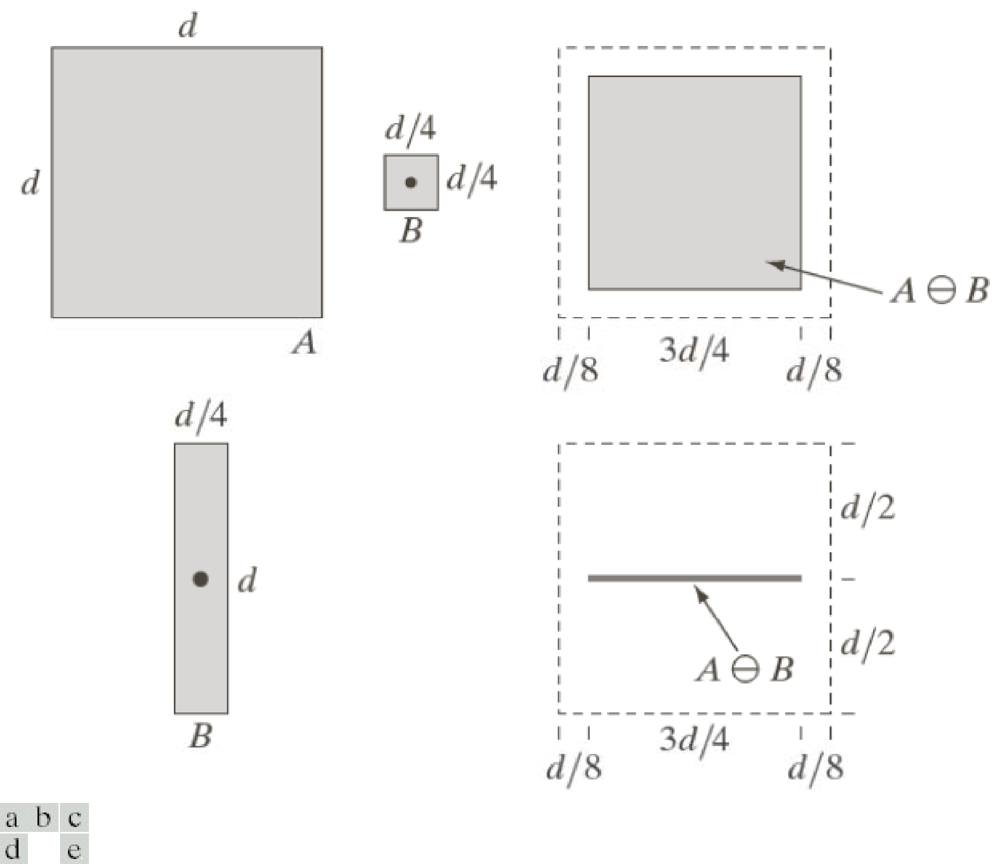



**FIGURE 9.6** (a) Set A. (b) Square structuring element. (c) Erosion of $A$ by $B$, shown shaded. (d) Elongated structuring element. (e) Erosion of $A$ using this element.

# Dilation and Erosion

使用13x13的方形結構，對圖(a)進行erosion

使用13x13的方形結構，對圖(b)進行dilation

Example of erosion
-eliminating irrelevant detail

a b c

**FIGURE 9.7** (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Example of erosion
-eliminating irrelevant detail

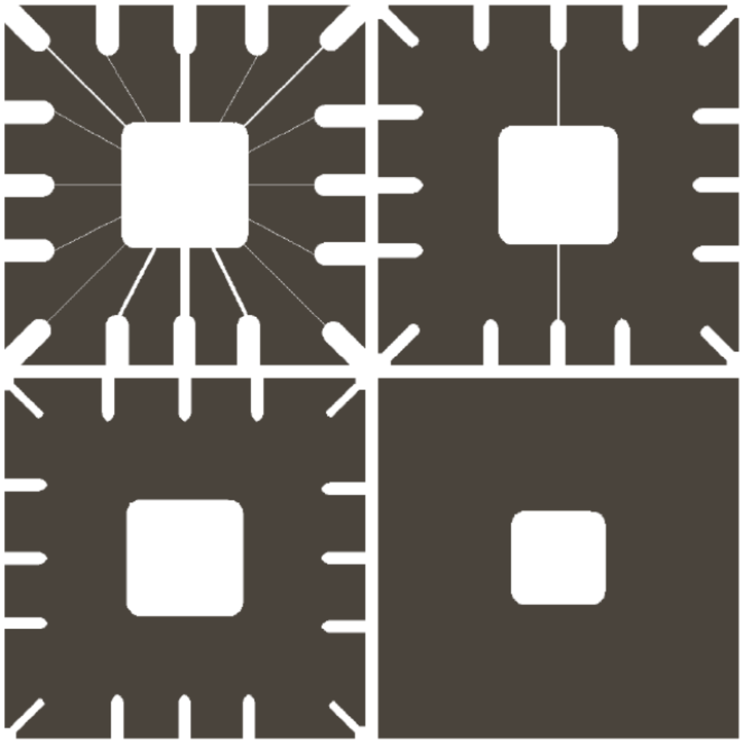


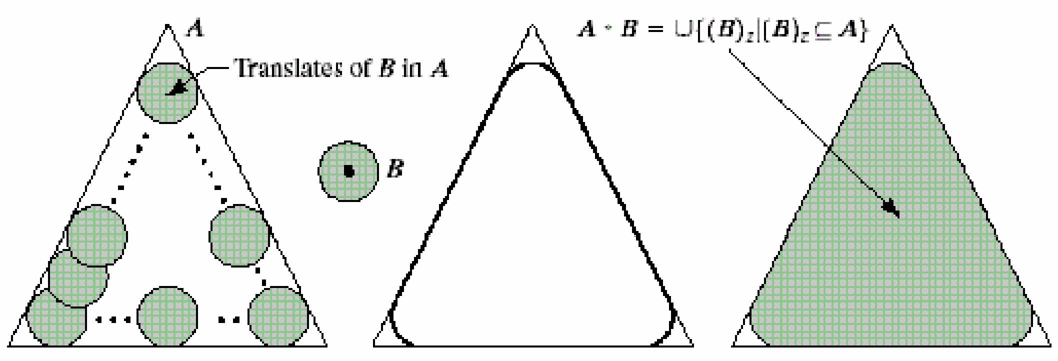

**FIGURE 9.5** Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

# Opening and Closing

- Opening
  - Generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions.

$$A \circ B = (A \ominus B) \oplus B$$
$$= \cup\{(B)_z \mid (B)_z \subseteq A\}$$

  - The opening $A$ by $B$ is the erosion of $A$ by $B$, followed by a dilation of the result by $B$.
    - View the structuring element $B$ as a flat "rolling ball"
    - The boundary of $A \circ B$ is then established by the points in $B$ that reach the farthest into the boundary of $A$ as $B$ is rolled around the inside of this boundary.

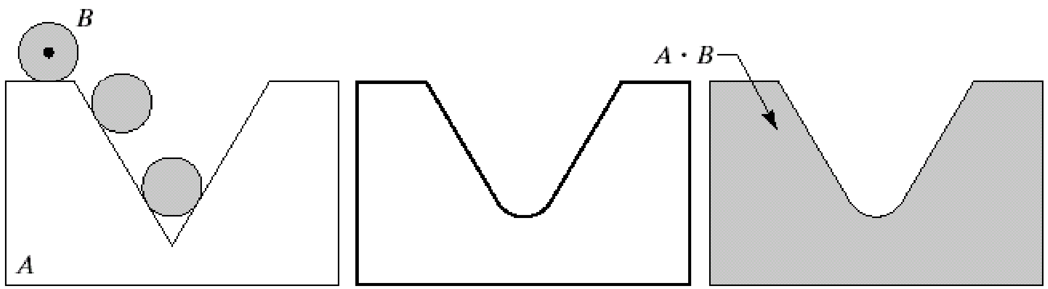


# Opening and Closing

- ***Closing***
  - Tends to smooth sections of contours, fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.
  - The closing of set $A$ by structuring element $B$, denoted $A \bullet B$

$$A \bullet B = (A \oplus B) \ominus B$$

  - The closing of $A$ by $B$ is simply the dilation of $A$ by $B$, followed by the erosion of the result by $B$.




a b c

**FIGURE 9.9** (a) Structuring element $B$ "rolling" on the outer boundary of set $A$. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

# Opening and Closing

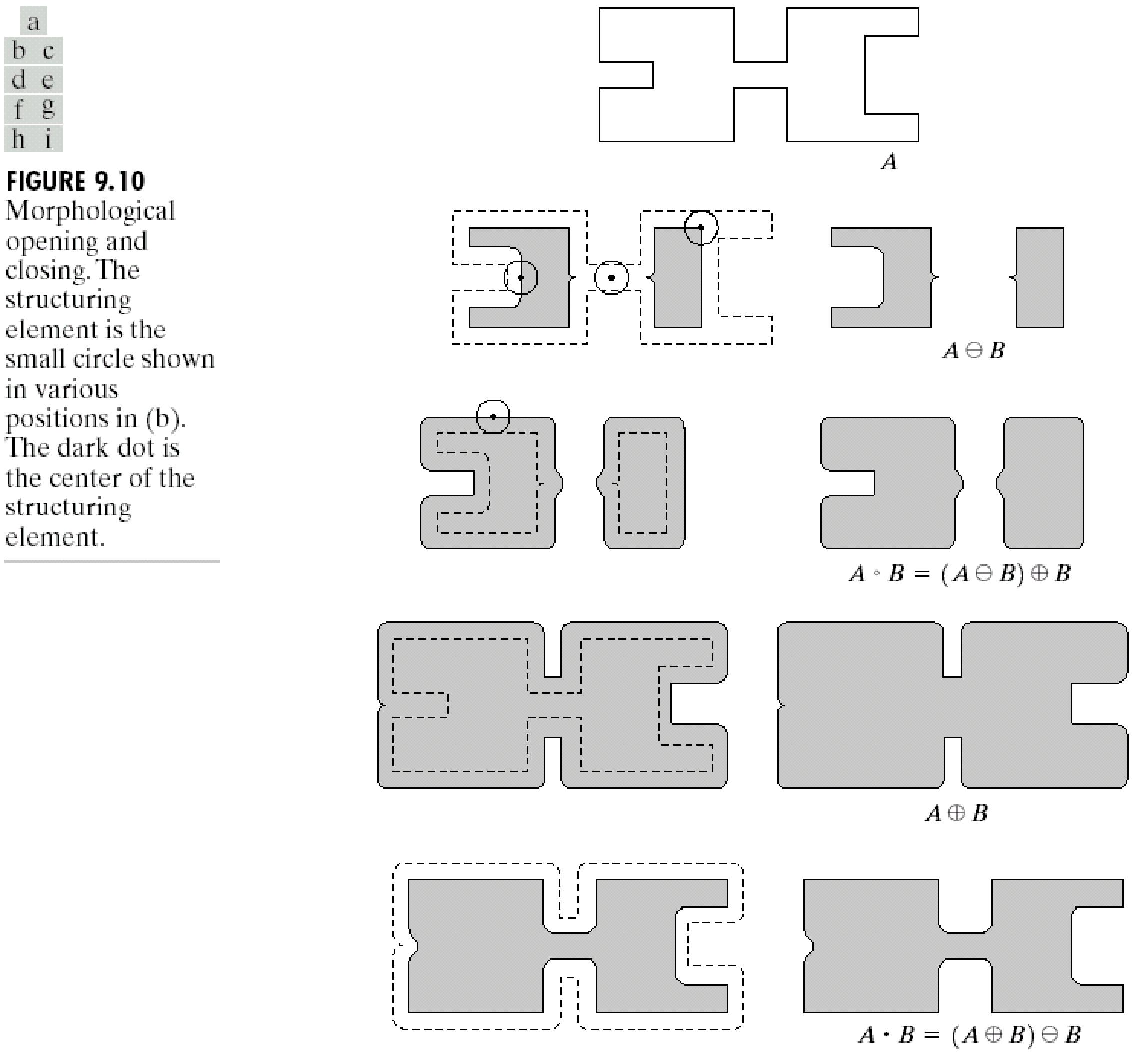



**FIGURE 9.10** Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

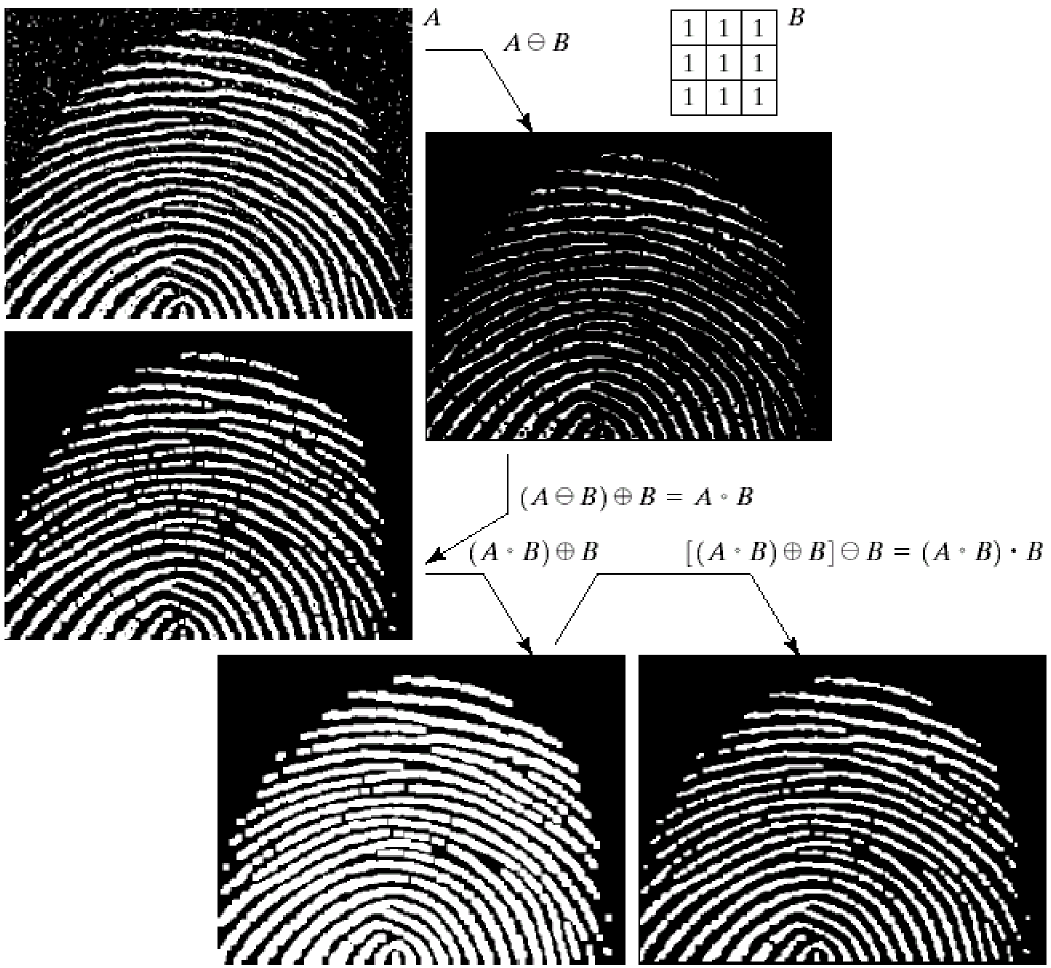
# Opening and Closing

- The opening operation satisfies the following properties
  - $A \circ B$ is a subset of $A$
  - If $C$ is a subset of $D$, then $C \circ B$ is a subset of $D \circ B$
  - $(A \circ B) \circ B = A \circ B$

# Opening and Closing

- The properties of closing operation
  - $A$ is a subset of $A \bullet B$
  - If $C$ is a subset of $D$, then $C \bullet B$ is a subset of $D \bullet B$
  - $(A \bullet B) \bullet B = A \bullet B$
  - Multiple openings or closings of a set have no effect after the operator has been applied once.

# Opening and Closing




a b c d e f

**FIGURE 9.11**
(a) Noisy image. (c) Eroded image. (d) Opening of $A$. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

# The Hit-or-Miss Transformation

- The Hit-or-Miss Transformation
  - The morphological hit-or-miss transform is a tool for shape detection.
  - If $B$ denotes the set composed of $A$ and its background, the match of $B$ in $A$, denoted $A \circledast B$ , is

在$A$中找$B$

$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W-X)] \qquad (9.4\text{-}1)$$

  - Let $B=(B_1, B_2)$, where $B_1$ is the set formed from elements of $B$ associated with an object and $B_2$ is the set of elements of $B$ associated with the corresponding background.

令$B=(B_1, B_2)$
$B_1$為欲detect的object
$B_2$為background

  - Let $B_1 = X$ and $B_2 = (W-X)$, Eq. (9.4-1) becomes

$$A \circledast B = (A \ominus B_1) \cap [A^c \ominus B_2] \qquad (9.4\text{-}2)$$

  - Thus, set $A \circledast B$ contains all the (origin) points at which, simultaneously, $B_1$ founded a match ("hit") in $A$ and $B_2$ found a match in $A^c$.

# The Hit-or-Miss Transformation

- By using the definition of *set differences* given in Eq(9.1-8) and the relationship between erosion and dilation given in Eq.(9.2-4), we can write Eq.(9.4-2) as

$$A \circledast B = (A \ominus B_1) - (A \oplus \overline{B}_2) \qquad (9.4\text{-}3)$$

- We refer to any of the preceding three equations as the *morphological hit-or-miss transformation*.

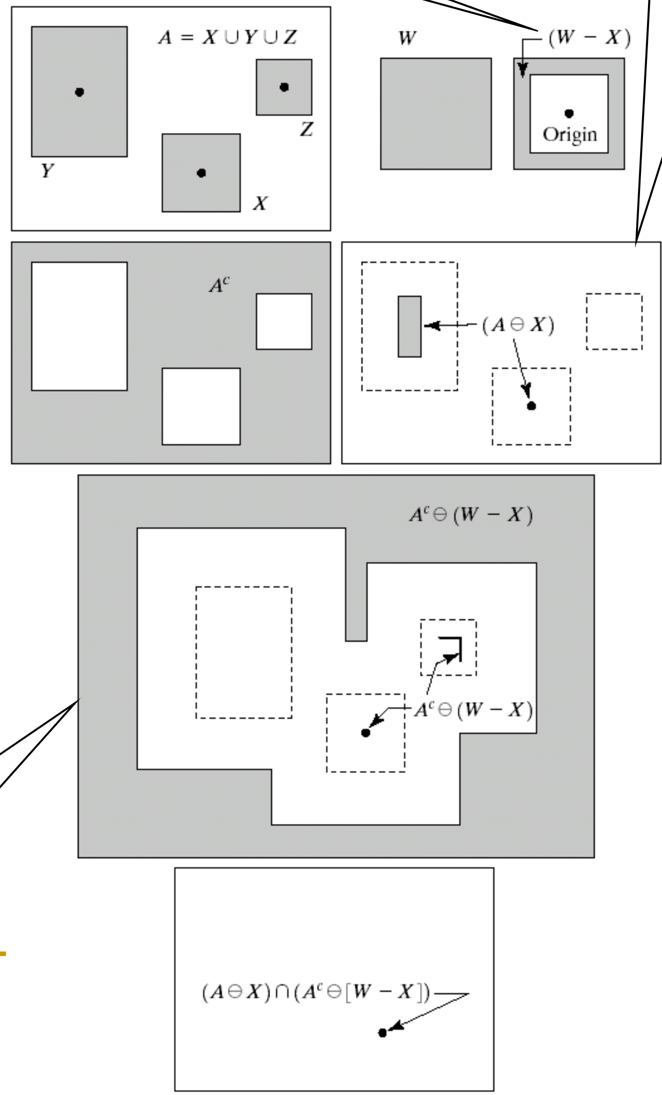
# The Hit-or-Miss Transformation

- The objective is to find the location of one of the shapes, say, $X$.
  - Let the origin of each shape be located at its center of gravity.
  - Let $X$ be enclosed by a small window, $W$.
  - The local background of $X$ with respect to $W$ is defined as the et difference ($W$-$X$), as Fig. (b).
  - Fig. (c) shows the complement of $A$
  - Fig. (d) shows the erosion of $A$ by $X$.
  - $A \ominus X$ may be viewed geometrically as the set of all locations of the origin $X$ at which $X$ found a match (hit) in $A$.
  - Fig. (e) shows the erosion of the complement of $A$ by the local background set ($W$-$X$).
  - From Fig. (d) and (e), the set of locations for which $X$ exactly fits inside $A$ is the intersection of the erosion of $A$ by $X$ and the erosion of $A^c$ by ($W$-$X$) as shown in Fig. (f).

將$X$用$W$包起來

$A$經$X$ erosion

$A^C$經($W$-$X$) erosion

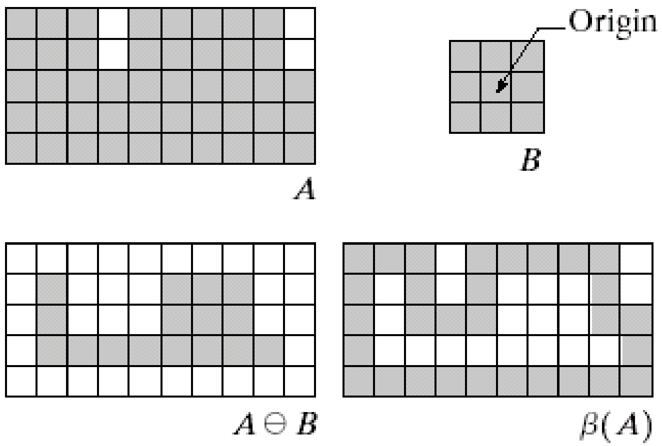


# Some basic Morphological Algorithm

- Boundary extraction
  - The boundary of a set $A$ can be obtained by first eroding $A$ by $B$ and then performing the set difference between $A$ and its erosion

$$\beta(A) = A - (A \ominus B)$$

  - Using a 5x5 structuring element of 1's would result in a boundary between 2 and 3 pixels thick.
  - When the origin of $B$ is on the edges of the set, part of the structuring element may be outside the image.
    - Assume that the values outside the borders of the image are 0.

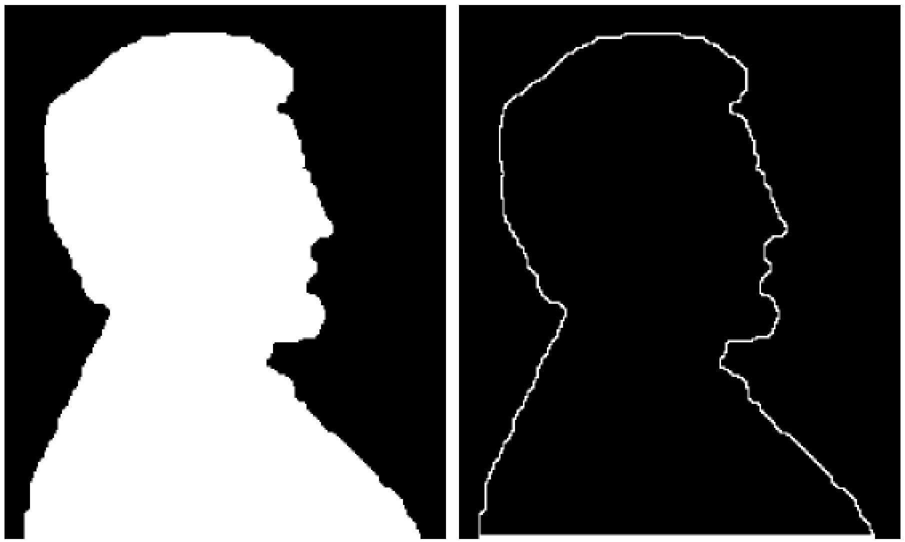



a b
c d

**FIGURE 9.13** (a) Set $A$. (b) Structuring element $B$. (c) $A$ eroded by $B$. (d) Boundary, given by the set difference between $A$ and its erosion.

# Some basic Morphological Algorithm

- Fig. 9.14
  - The structuring element as shown in Fig. 9.13(b).
  - Binary 1's are shown in white and 0's in black.
  - The boundary shown in Fig. 9.14(b) is one pixel thick.



a b

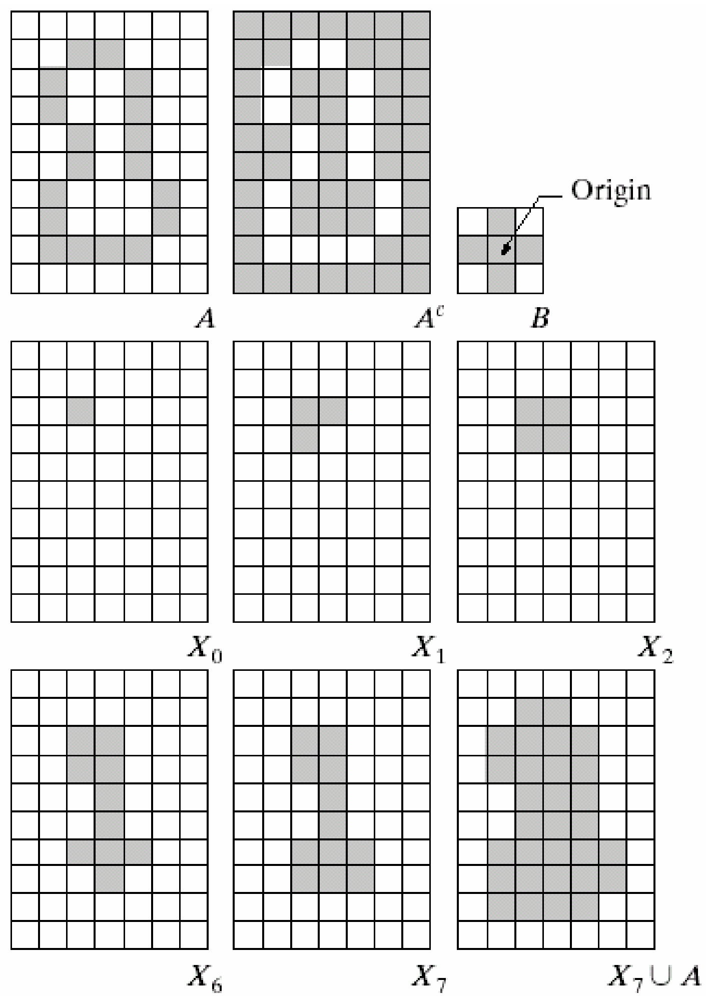
**FIGURE 9.14** (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

# Some basic Morphological Algorithm

- Region Filling
  - In Fig. 9.15, $A$ denotes a set containing a subset whose elements are 8-connected boundary points of a region.
  - Beginning with a point $p$ inside the boundary, the objective is to fill the entire region with 1's.
  - Assume that all non-boundary points are labeled 0.
  - Assign a value of 1 to $p$ to begin.
  - The following procedure then fills the region with 1's:

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad k = 1,2,3,\ldots$$

  - where $X_0 = p$, and $B$ is the symmetric structuring elements shown in Fig. 9.15(c).
  - The algorithm terminates at iteration step $k$ if $X_k = X_{k-1}$.
  - The set union of $X_k$ and $A$ contains the filled set and its boundary.



# Some basic Morphological Algorithm

- Example 9.6: Morphological region filling
  - An image composed of white circles with black inner spots.
  - The objective is to eliminate the reflections by region filling.

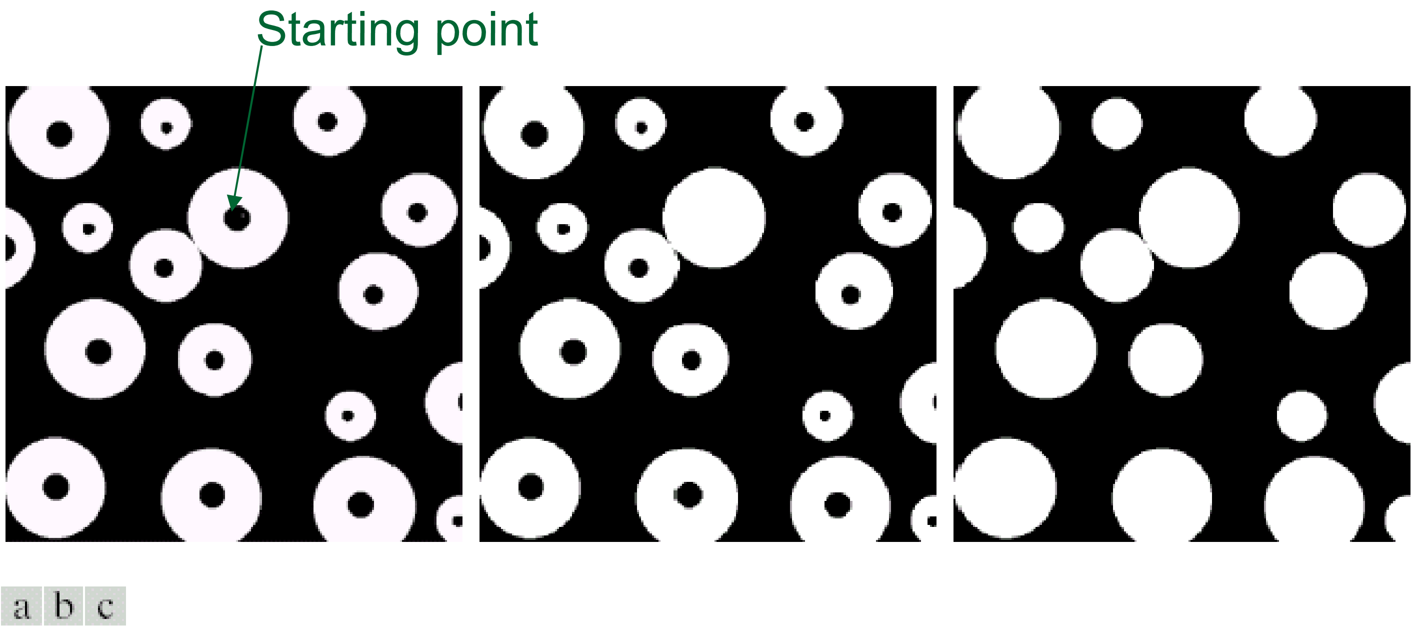



**FIGURE 9.16** (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

# Some basic Morphological Algorithm

- Extraction of Connected Components
  - Let $Y$ represent a connected component contained in a set $A$ and assume that a point $p$ of $Y$ is known.
  - The following iterative expression yields all the elements of $Y$:

$$X_k = (X_{k-1} \oplus B) \cap A \qquad k = 1,2,3,\ldots$$

  - where $X_0 = p$, and $B$ is a suitable structuring element, as shown in Fig. 9.17.
  - If $X_k = X_{k-1}$, the algorithm has converged and we let $Y = X_k$.

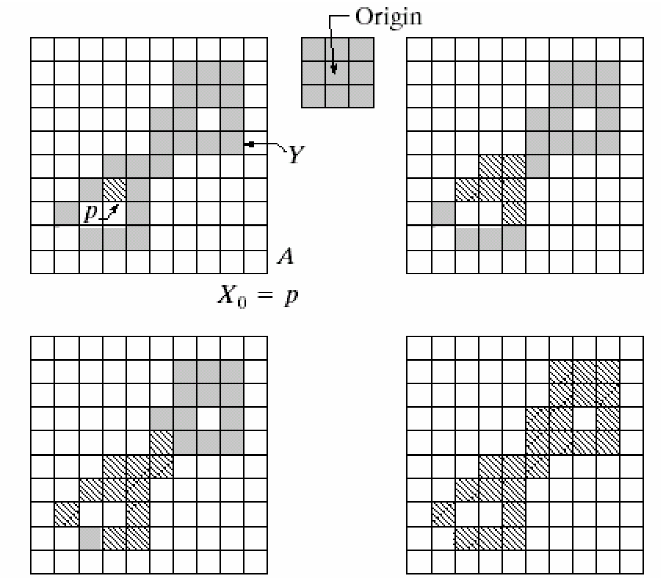



**FIGURE 9.17** (a) Set $A$ showing initial point $p$ (all shaded points are valued 1, but are shown different from $p$ to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

# Some basic Morphological Algorithm

- Example 9.7:
  - Used for automated inspection.
  - Fig. 9.18(a) shows an X-ray image of a chicken breast that contains bone fragments.
    - To detect foreign objects in processed food before packaging or shipping.

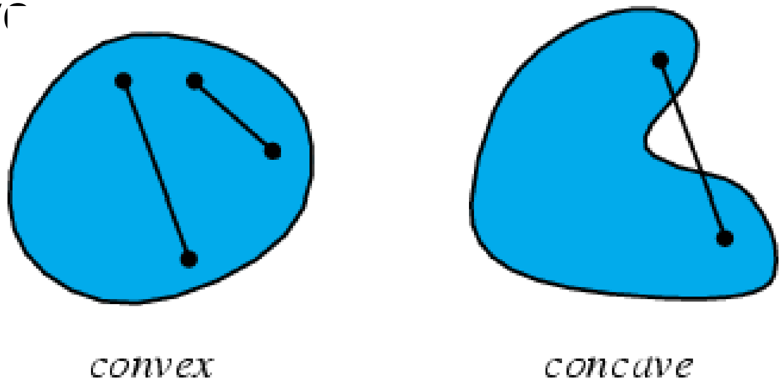
a
b
c d

**FIGURE 9.18** (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

| Connected component | No. of pixels in connected comp |
|---|---|
| 01 | 11 |
| 02 | 9 |
| 03 | 9 |
| 04 | 39 |
| 05 | 133 |
| 06 | 1 |
| 07 | 1 |
| 08 | 743 |
| 09 | 7 |
| 10 | 11 |
| 11 | 11 |
| 12 | 9 |
| 13 | 9 |
| 14 | 674 |
| 15 | 85 |

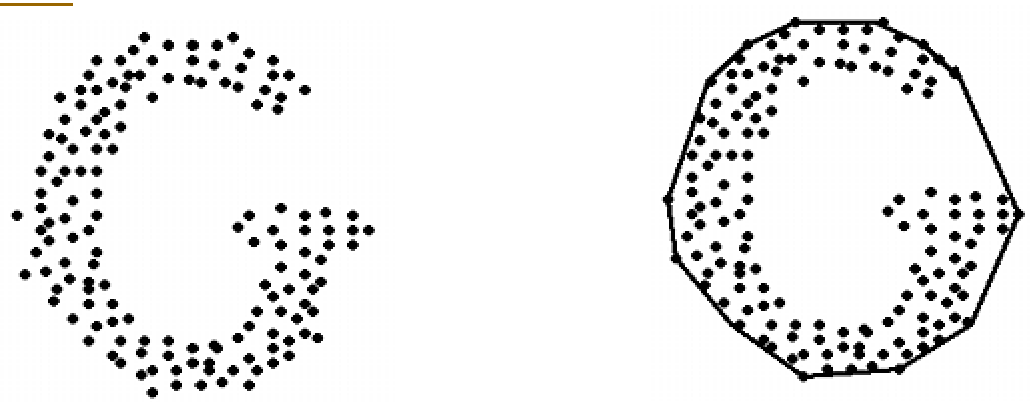
# Convex sets

- Convex sets
  - A convex set is a set of elements from a vector space such that all the points on the straight line between any two points of the set are also contained in the set.
  - A set S in n-dimensional space is called a convex set if the line segment joining any pair of points of S lies entirely in S. If the set does not contain all the line segments, it is called concave.



# Convex sets

- Convex Hull
  - The convex hull of a set of points is the smallest convex set that includes the points. For a two dimensional finite set the convex hull is a convex polygon.
  - http://www.cse.unsw.edu.au/~lambert/java/3d/ConvexHull.html



# Convex sets

- Convex hull 演算法一: Jarvis's March (gift wrapping)
  - 找出最下方的點 $p_0$. 它一定在 convex hull 的邊界上.
  - 找出 $p_1$, 使 $p_0$ 與 $p_1$ 的連線與正 x 軸的夾角 (有向角) 最小.
  - 找出 $p_2$, 使 $p_2$ 與 $p_1$ 的連線與正 x 軸的夾角最小.
  - ... 直到回到 $p_0$ 為止.

# Convex sets

- Convex hull 演算法二: Graham's scan
  - 找出最下方的點 $p_0$. 它一定在 convex hull 的邊界上.
  - 以「$p_0$ 到各點的射線與 x 軸的夾角」作為比較的依據, 對所有的點排序.
  - 依序如下檢查 $p_1$, $p_2$,....
  - 檢查 $p_i$ 時要做的事情: 看看 stack 上第二高的元素, stack 上最頂端的元素, 與 $p_i$ 三點兩射線是左轉還是右轉. 如果是右轉, 就 pop, 並重複此步驟.
  - push $p_i$.
  - 檢查下一個 $p_i$.

# Some basic Morphological Algorithm

- Convex Hull
  - A set $A$ is said to be convex if the straight line segment jointing any two points in $A$ lies entirely within $A$.
  - The *convex hull* $H$ of an arbitrary set $S$ is the smallest convex set containing $S$.
  - The set difference $H$-$S$ is called the *convex deficiency* of $S$.
  - The convex hull and convex deficiency are useful for object description.
  - Let $B^i$, $i$=1, 2, 3, 4, represent the four structuring elements in Fig. 9.19 (a).
  - The procedure consists of implementing the equation:

$$X_k^i = \left(X_{k-1} \otimes B^i\right) \cup A \qquad i = 1,2,3,4 \text{ and } k = 1,2,3,\ldots$$

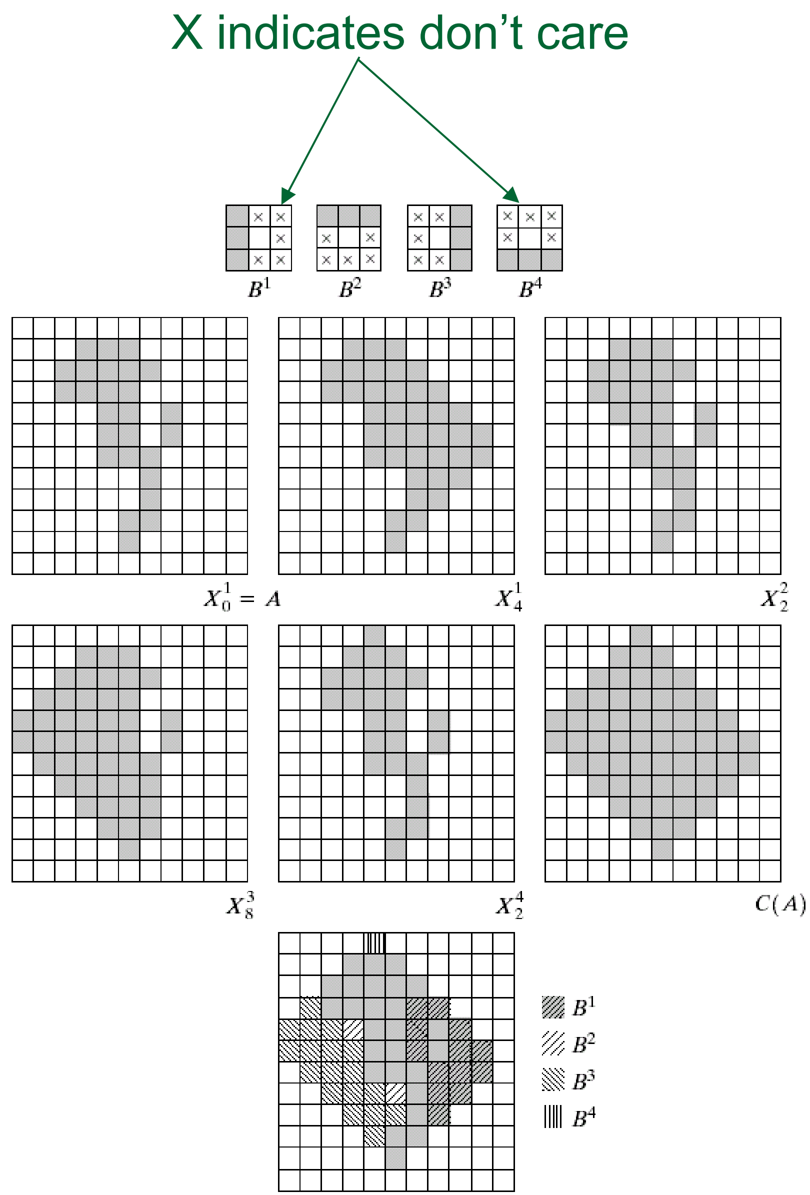
    where $X_0^i = A$

  - Let $D^i = X_{conv}^i$. Then the convex hull of $A$ is

$$C(A) = \bigcup_{i=1}^{4} D^i$$

# Some basic Morphological Algorithm

- The procedure consists of iteratively applying the hit-or-miss transform to $A$ with $B^1$; when no further changes occur, we perform the union with $A$ and call the result $D^1$.
- The procedure is repeated with $B^2$ until no further changes occur, and so on.
- The union of the four resulting $D$'s constitutes the convex hull of $A$.




a
b c d
e f g
h

**FIGURE 9.19** (a) Structuring elements. (b) Set $A$. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

# Some basic Morphological Algorithm

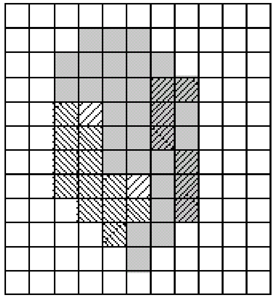


**FIGURE 9.20** Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

# Some basic Morphological Algorithm

- The thinning of a set $A$ by a structuring element $B$, denoted by $A \otimes B$, can be defined by the hit-or-miss transform:

  $$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$

- A more useful expression for thinning $A$ symmetrically is based on a sequence of structuring elements:

  $$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

  where $B^i$ is a rotated version of $B^{i-1}$
- Based on the concept, the thinning can be defined by a sequence of structuring element

  $$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

- The process is to thin $A$ by one pass with $B^1$, then thin the result with one pass of $B^2$, and so on, until $A$ is thinned with one pass of $B^n$.
- The entire process is repeated until no further changes occur.

# Some basic Morphological Algorithm

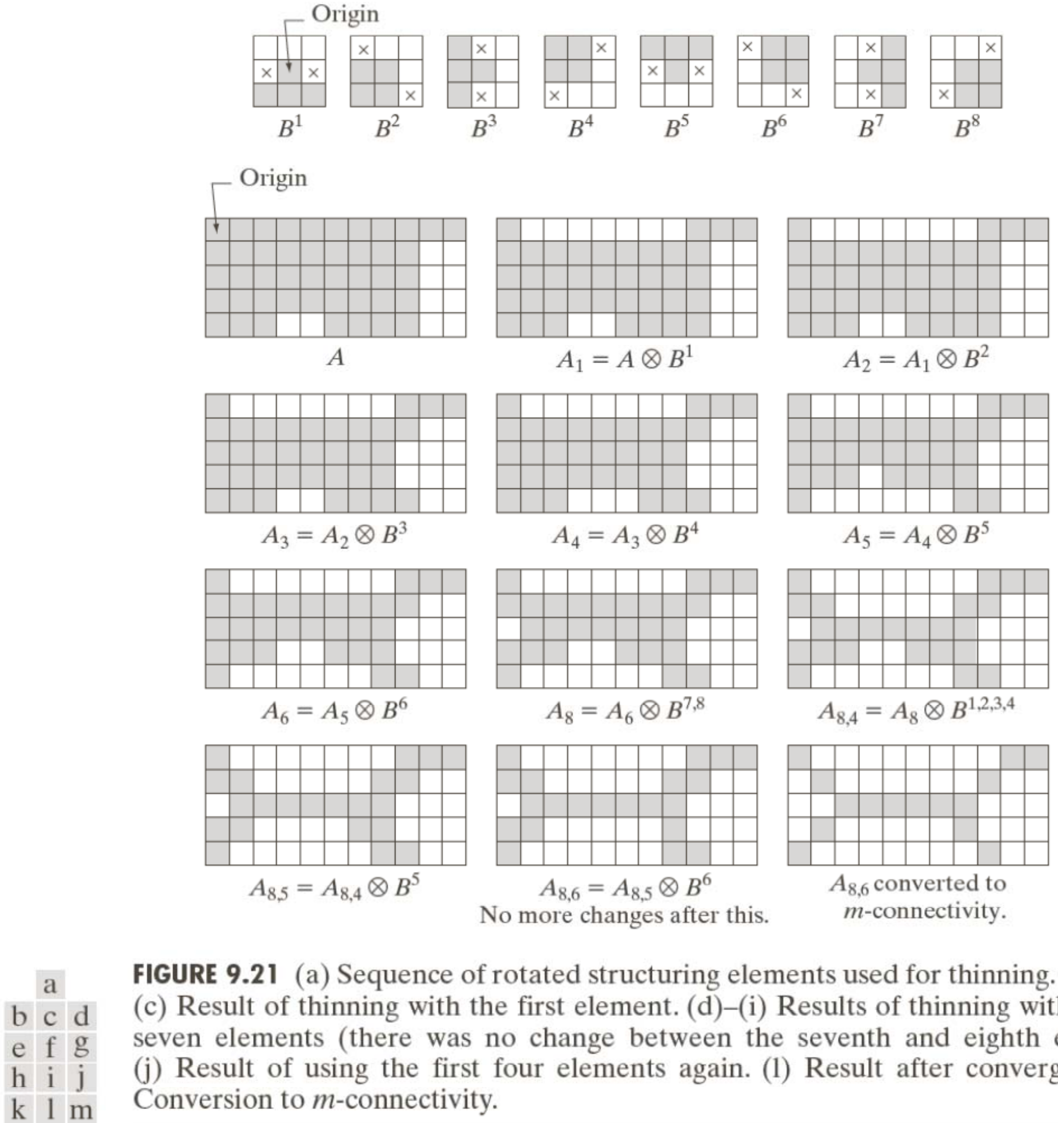



**FIGURE 9.21** (a) Sequence of rotated structuring elements used for thinning. (b) Set $A$. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to $m$-connectivity.

# Some basic Morphological Algorithm (cont.)

- Thickening
  - Thickening is the morphological dual of thinning and is defined by

    $$A \odot B = A \cup (A \circledast B)$$

    where $B$ is a structuring element suitable for thickening.
  - As in thinning , thickening can be defined as a sequential operation:

    $$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2) \cdots) \odot B^n)$$

  - The structuring elements used for thickening have the same form as those shown in Fig. 9.21(a) in connection with thinning, but with all 1's and 0's interchanged.
  - The usual procedure is to thin the background of the set in question and then complement the result.(欲使某集合變粗，可細化該集合的背景後，取其背景的補集。)
    - Thicken a set $A$, we form $C=A^c$, thin $C$, and then form $C^c$.

# Some basic Morphological Algorithm

- **Thickening by background thinning**
  - Thicken a set $A$, we form $C=A^c$, thin $C$, and then form $C^c$.

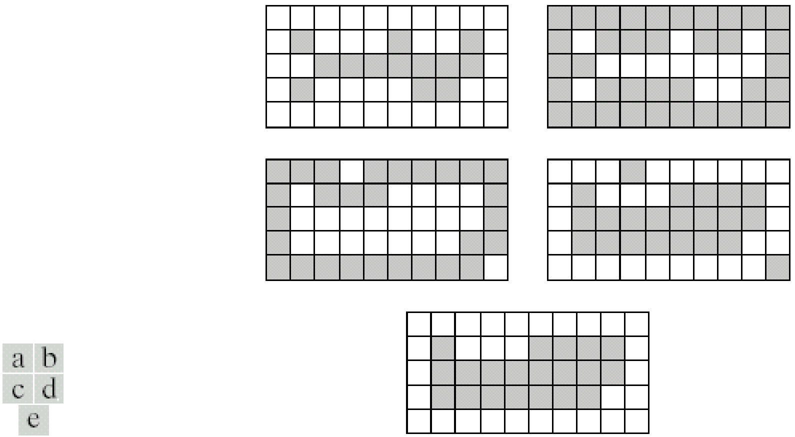


**FIGURE 9.22** (a) Set $A$. (b) Complement of $A$. (c) Result of thinning the complement of $A$. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

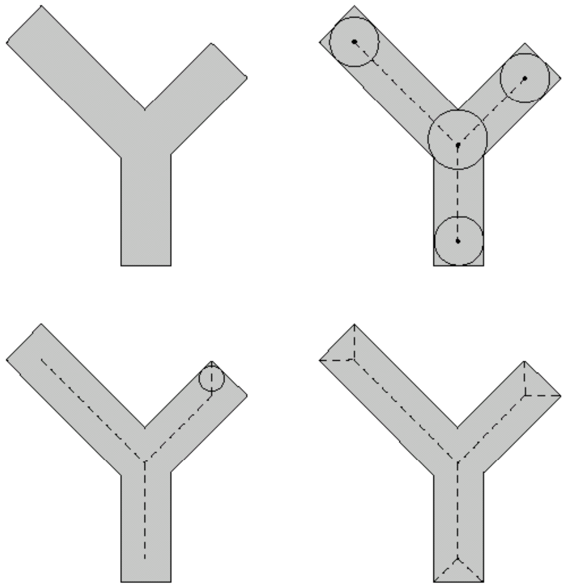
- 欲使某集合變粗，可細化該集合的背景後，取其背景的補集。
- 欲粗化集合$A$的步驟：
  - 先求$A$的背景，$C=A^c$
  - 細化 $C$
  - 求$C$的補集，$C^c$

# Some basic Morphological Algorithm (cont.)

- Skeletons
  - As shown in Fig. 9.23, the notation of a skeleton, $S(A)$, of a set $A$ is intuitively simple. We deduce from this figure that
    - If $z$ is a point of $S(A)$ and $(D)_z$ is the largest disk centered at $z$ and contained in $A$, one cannot find a larger disk containing $(D)_z$ and included in $A$. The disk $(D)_z$ is called a *maximum disk*.
    - The disk $(D)_z$ touches the boundary of $A$ at two or more different places.

a b
c d

**FIGURE 9.23**
(a) Set $A$.
(b) Various positions of maximum disks with centers on the skeleton of $A$.
(c) Another maximum disk on a different segment of the skeleton of $A$.
(d) Complete skeleton.



# Some basic Morphological Algorithm (cont.)

- Skeletons
  - The skeleton of $A$ can be expressed in terms of erosions and openings:

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

where $B$ is a structuring element, and $(A \ominus kB)$ indicates $k$ successive erosions of $A$:

$$(A \ominus kB) = (...(A \ominus B) \ominus B) \ominus ...) \ominus B$$

表示$A$經過$k$次$B$的侵蝕

# Some basic Morphological Algorithm (cont.)

- $K$ is the last iterative step before $A$ erodes to an empty set. In other words, ($A$被$B$侵蝕成空集合前的最後一次侵蝕)

$$K = \max\{k \mid (A \ominus kB) \neq \phi\}$$

- The formulation given in Eqs.(9.5-11) and (9.5-12) states that $S(A)$ can be obtained as the union of the skeleton subsets $S_k(A)$.
- $A$ can be reconstructed from these subsets by using the equation.(集合$A$可由其骨架子集合$S_k(A)$ ，經過$k$次膨脹(dilation)後復原。)

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where $(Sk(A) \oplus kB)$ denotes $k$ successive dilations of $S_k(A)$

$$(S_k(A) \oplus kB) = ((\ldots(S_k(A) \oplus B) \oplus B) \oplus \ldots) \oplus B$$

# Basic morphological Algorithm

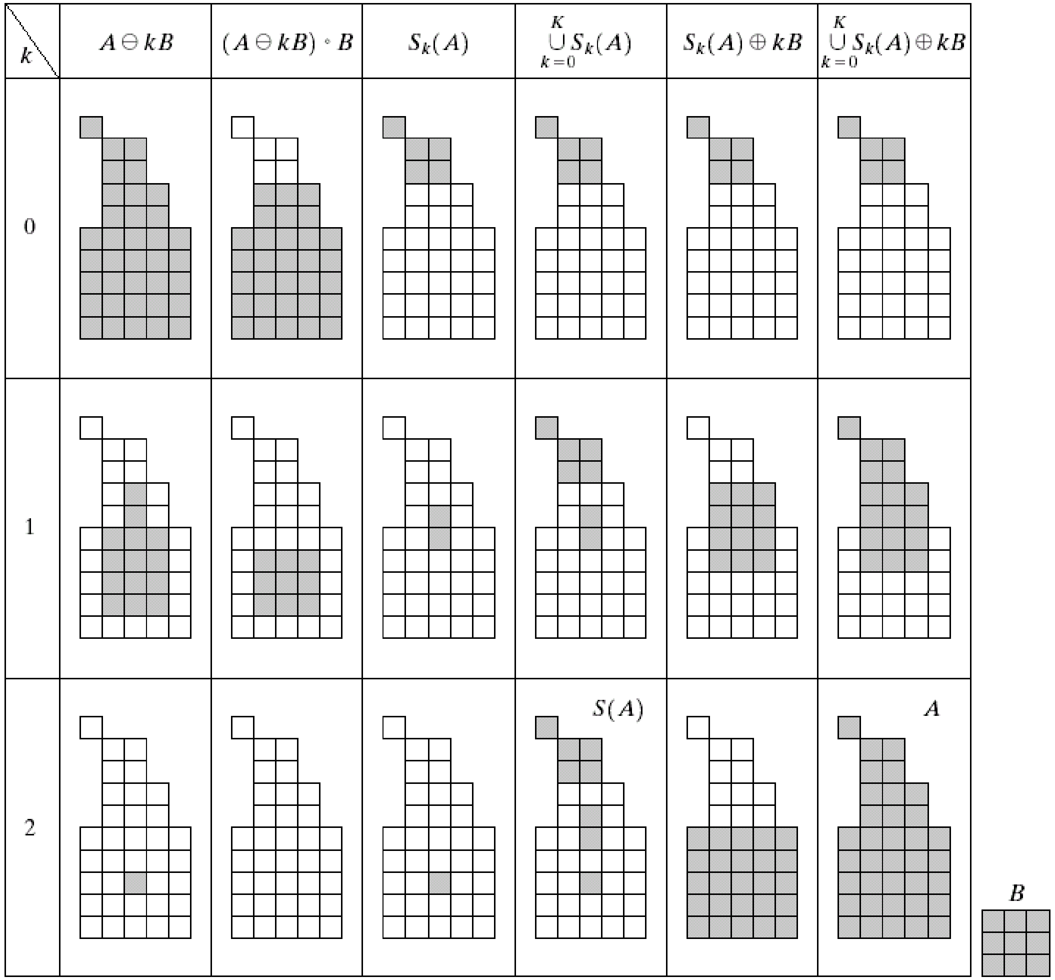
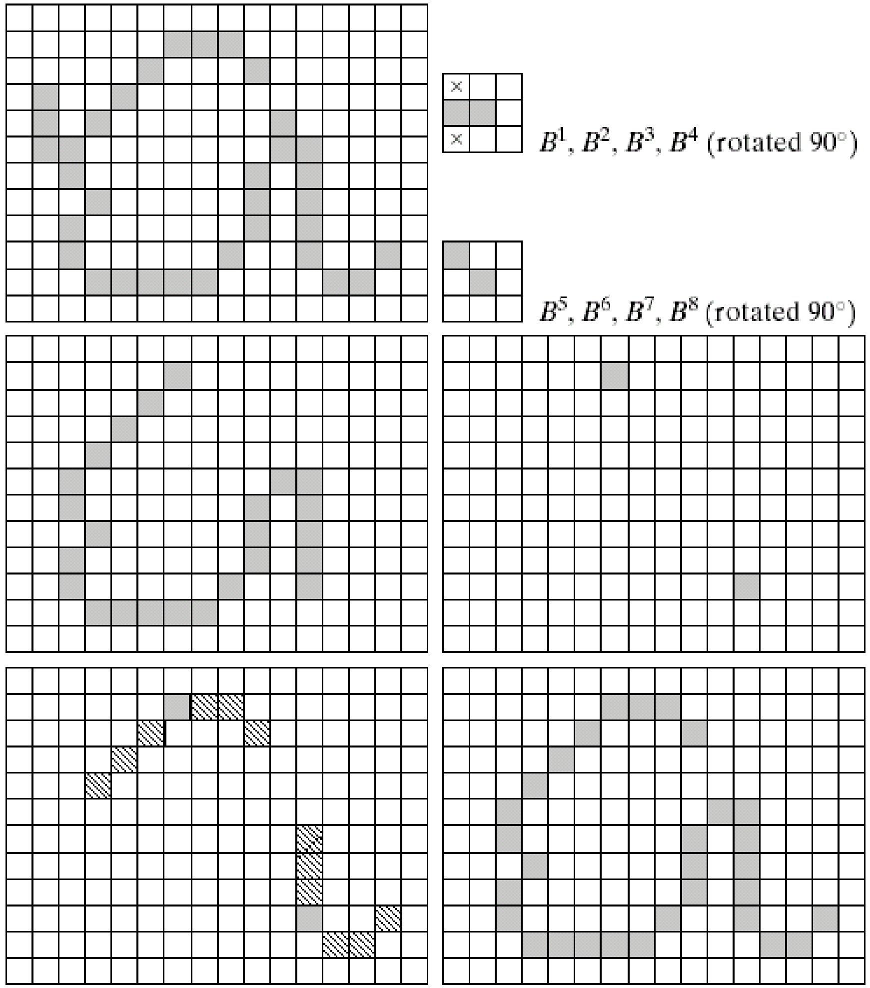



**FIGURE 9.24** Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

# Basic morphological Algorithm

- 修剪(Pruning)
  - 由於細化及骨架化後，常會遺留一些小分支，需清除，因此需要修剪演算法。
  - 基本觀念：
    - 假設parasitic component的長度不超過一特定值，pruning的做法在於藉由連續的刪減端點(end point) ，來減少寄生分枝。
    - 作法：
      - 將集合$A$ 經過結構元素$B$的細化，$X_1 = A \otimes \{B\}$
      - 找出 $A$的end point，$X_2 = \bigcup_{k=1}^{8} \left( X_1 \otimes B^k \right)$
      - 對$X_2$進行dilation，$X_3 = (X_2 \oplus H) \cap A$
      - 取$X_1$和 $X_3$的聯集，$X_4 = X_1 \cup X_3$

# Basic morphological Algorithm (cont.)




a b
c
d e
f g

**FIGURE 9.25** (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

# Summary of Morphological Operations on Binary Images

**TABLE 9.2**
Summary of morphological operations and their properties.

| Operation | Equation | Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26). |
|---|---|---|
| Translation | $(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$ | Translates the origin of $A$ to point $z$. |
| Reflection | $\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$ | Reflects all elements of $B$ about the origin of this set. |
| Complement | $A^c = \{w \mid w \notin A\}$ | Set of points not in $A$. |
| Difference | $A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$ | Set of points that belong to $A$ but not to $B$. |
| Dilation | $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$ | "Expands" the boundary of $A$. (I) |
| Erosion | $A \ominus B = \{z \mid (B)_z \subseteq A\}$ | "Contracts" the boundary of $A$. (I) |
| Opening | $A \circ B = (A \ominus B) \oplus B$ | Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I) |
| Closing | $A \bullet B = (A \oplus B) \ominus B$ | Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I) |

# Summary of Morphological Operations on Binary Images (cont.)

| | | |
|---|---|---|
| Hit-or-miss transform | $A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$ | The set of points (coordinates) at which, simultaneously, $B_1$ found a match ("hit") in $A$ and $B_2$ found a match in $A^c$. |
| Boundary extraction | $\beta(A) = A - (A \ominus B)$ | Set of points on the boundary of set $A$. (I) |
| Region filling | $X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$ | Fills a region in $A$, given a point $p$ in the region. (II) |
| Connected components | $X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$ | Finds a connected component $Y$ in $A$, given a point $p$ in $Y$. (I) |
| Convex hull | $X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$. | Finds the convex hull $C(A)$ of set $A$, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III) |

**TABLE 9.2**
Summary of morphological results and their properties.
*(continued)*

# Reference


- Rafael C. Gonzalez and Richard E. Woods, Digital Image Processing (2nd, 3rd Edition)
- Atam P. Dhawan, Medical Image Analysis, Wiley Interscience, 2003.